DELFT UNIVERSITY OF TECHNOLOGY
Department of civil engineering
Division of sanitary engineering

Ground-Water Recovery

Problems and their solution

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G R O U N D W A T E R   R E C O V E R Y

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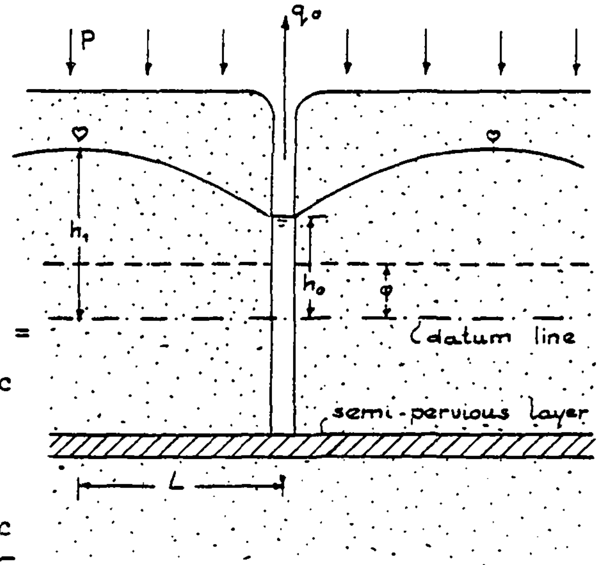
1.01 An unconfined aquifer is situated above a semi-pervious layer, below which artesian water at a constant and uniform pressure rising to 0.6 m above datum line is present. The unconfined aquifer is recharged by rainfall in an amount of $(40)10^{-9}$ m/sec and crossed by a ditch, abstracting groundwater at a rate of $(35)10^{-6}$ m^3/m'/sec. Due to this abstraction a cone of depression is formed, extending 750 m at either side of the ditch, with waterlevels of 2.5 m above datum line at the water divides and 1.3 m above datum line at the ditch.

What is the resistance of the semi-pervious layer against vertical water movement?



The water balance for the strip of land between both water divides reads

recharge $2PL = (2)(40)10^{-9}(750) =$
$= (60)10^{-6}$ m^3/m'/sec

abstraction $q_o =$ $(35)10^{-6}$ m^3/m'/sec

downward percolation
$I = (25)10^{-6}$ m^3/m'/sec

On the other hand, this downward percolation equals

$$I = 2L \frac{h_a - \phi}{c}$$

with c as resistance of the semi-pervious layer against vertical water movement and h_a as average water table depth in the unconfined aquifer. Assuming this water table to have a parabolic shape gives

$$h_a = h_o + \frac{2}{3}(h_1 - h_o) = 1.3 + \frac{2}{3}(2.5 - 1.3) = 2.1 \text{ m}$$

and $$c = \frac{2L(h_a - \phi)}{I} = \frac{(2)(750)(2.1 - 0.6)}{(25)10^{-6}} = (84)10^6 \text{ sec}$$

1.02 An unconfined aquifer is situated above a semi-pervious layer, below which artesian water at a constant and uniform level ϕ is present. In the unconfined aquifer an area A is considered, composed of two equal parts A_1 and A_2 over which the resistance of the semi-pervious layer against vertical water movement amounts to $(30)10^6$ and $(90)10^6$ sec respectively.

Calculate the resistance of the semi-pervious layer as average over the full area A

a. in case the phreatic water table h in the unconfined aquifer is constant at 1.5 m above the artesian water table;
b. in case the water table depth h_1 is augmented to 4.5 m and the water table depth h_2 remains unchanged.

The resistance c of the semi-pervious layer as average over the full area A is defined by the formula

$$Q = A\,\frac{h - \phi}{c}$$

with Q as amount of downward percolating water. This amount must equal the sum of the downward water movement over both halves

$$A\,\frac{h - \phi}{c} = \frac{A}{2}\,\frac{h_1 - \phi_1}{c_1} + \frac{A}{2}\,\frac{h_2 - \phi_2}{c_2}$$

From this equation A falls out, giving as relations

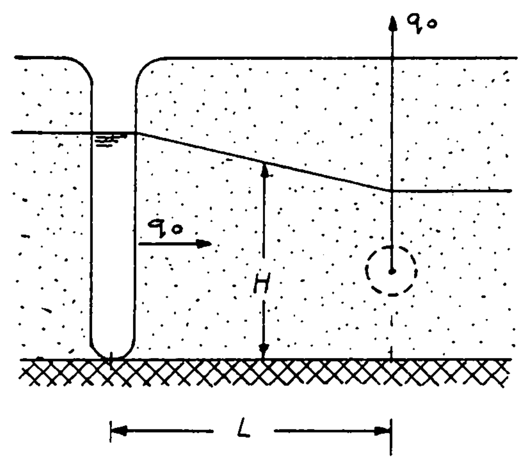
a. $\frac{1.5}{c} = \frac{1}{2}\,\frac{1.5}{(30)10^6} + \frac{1}{2}\,\frac{1.5}{(90)10^6}$ or $c = (45)10^6$ sec

b. $\frac{0.5(1.5+4.5)}{c} = \frac{1}{2}\,\frac{4.5}{(30)10^6} + \frac{1}{2}\,\frac{1.5}{(90)10^6}$ or $c = (36)10^6$ sec

From this calculation the conclusion may be drawn that the average resistance of a semi-pervious layer against vertical water movement is <u>not</u> a geo-hydrological constant.

1.11 An unconfined aquifer is composed of sand with a specific yield μ of 35% and is situated above an impervious base. In this aquifer a gallery is constructed at a distance L parallel to a river. From the gallery, groundwater is abstracted in an amount of $(0.1)10^{-3}$ $m^3/m'/sec$, while an equal amount of riverwater is induced to recharge the aquifer. By this horizontal water movement, the saturated thickness of the aquifer drops from 18 m near the river to 16 m near the gallery.

Which distance L must be chosen to assure that the infiltrating riverwater stays at least 2 months in the sub-soil before it is recovered by the gallery?



With v as real velocity of horizontal water movement and T as required detention time, the minimum length of travel L equals

$$L = vT = \frac{q_o}{\mu H} T$$

in which H is the average saturated thickness. In the case under consideration

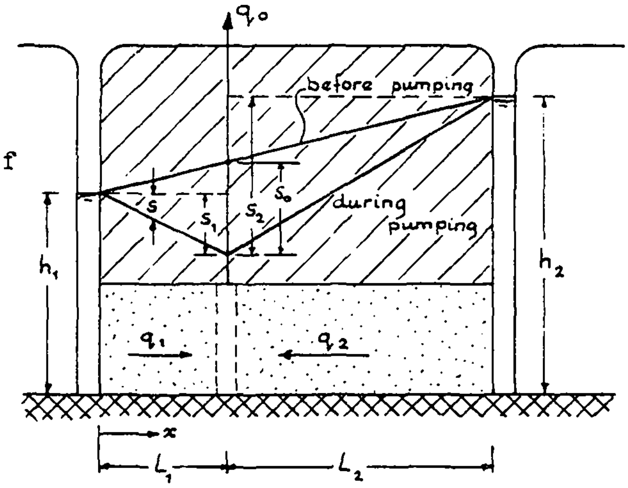
$$H = 0.5(18 + 16) = 17 \text{ m} \quad \text{With } T = 2 \text{ months} = (5.3)10^6 \text{ sec}$$

$$L = \frac{(0.1)10^{-3}}{(0.35)17} (5.3)10^6 = 90 \text{ m}$$

Taking into account the concentration of flowlines in the immediate vicinity of the gallery, a distance of 100 m should be chosen.

2.01 An artesian aquifer is situated above an impervious base and is overlain by an impervious aquiclude. The thickness H of the artesian aquifer is constant at 16 m, its coefficient of permeability k at $(0.60)10^{-3}$ m/sec. Two parallel fully penetrating ditches isolate in this aquifer a strip of land 2500 m wide. The water levels in the ditches are constant at 20 and 23 m above the impervious base respectively.

At a distance of 800 m parallel to the ditch with the lowest water table, a fully penetrating gallery is constructed. Determine the capacity-drawdown relationship for this gallery.



With the notations as indicated in the figure at the right, the equations of flow become

Darcy $q_1 = kH \frac{ds}{dx}$

continuity q_1 = constant

combined $ds = \frac{q_1}{kH} dx$

integrated between the limits $x = 0, s = 0$ and $x = L_1$, $s = s_1$

$$s_1 = \frac{q_1}{kH} L_1 \qquad \text{or} \qquad q_1 = kH \frac{s_1}{L_1}$$

similarly $s_2 = \frac{q_2}{kH} L_2$ or $q_2 = kH \frac{s_2}{L_2}$

$$q_o = q_1 + q_2 = kH(\frac{s_1}{L_1} + \frac{s_2}{L_2})$$

According to the figure

$$s_1 = s_o - \frac{L_1}{L_1 + L_2}(h_2 - h_1)$$

$$s_2 = s_o + \frac{L_2}{L_1 + L_2}(h_2 - h_1), \quad \text{substituted}$$

$$q_o = kH\,(\frac{s_o}{L_1} + \frac{s_o}{L_2})$$

With the data as given

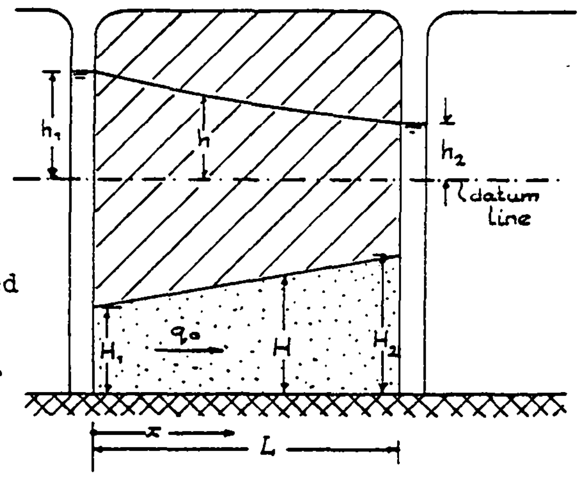
$$q_o = (0.60)10^{-3}\ (16)\left(\frac{1}{800} + \frac{1}{1700}\right)s_o$$

$$q_o = (1.77)10^{-6}\ s_o \quad m^3/m'/sec$$

2.02 An artesian aquifer without recharge from above or from below has a constant coefficient of permeability k of $(0.35)10^{-3}$ m/sec, but a thickness which varies linearly with distance. In this aquifer two fully penetrating ditches isolate a strip of land with a constant width of 4200 m. The water levels in these ditches are constant at 8 m and at 3 m above datum line, while the aquifer thickness at the face of the ditches amounts to 16 and 28 m respectively.

How much water flows through the aquifer from one ditch to the other?

Without recharge from above or from below, the rate of flow in the artesian aquifer is constant, equal to q_o. With the notations as indicated in the picture at the right, Darcy's law gives this rate of flow as



$$q_o = - kH \frac{dh}{dx} \qquad \text{with H variable with distance}$$

$$H = H_1 + \frac{H_2 - H_1}{L} x \qquad \text{substituted}$$

$$q_o = - k \left(H_1 + \frac{H_2 - H_1}{L} x\right) \frac{dh}{dx} \qquad \text{or}$$

$$dh = - \frac{q_o}{k} \frac{dx}{H_1 + \frac{H_2 - H_1}{L} x}$$

Integrated between the limits $x = 0$, $h = h_1$ and $x = L$, $h = h_2$

$$h \Big|_{h_1}^{h_2} = - \frac{q_o}{k} \frac{L}{H_2 - H_1} \ln \left(H_1 + \frac{H_2 - H_1}{L} x\right) \Big|_0^L$$

$$h_2 - h_1 = - \frac{q_o}{k} \frac{L}{H_2 - H_1} \ln \frac{H_2}{H_1}$$

$$q_o = k \frac{h_1 - h_2}{L} \frac{H_2 - H_1}{\ln \frac{H_2}{H_1}}$$ With the data supplied

$$q_o = (0.35)10^{-3} \frac{8 - 3}{4200} \frac{28 - 16}{\ln \frac{28}{16}} = (0.35)10^{-3} \frac{5}{4200} \frac{12}{0.56}$$ or

$$q_o = (8.9)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

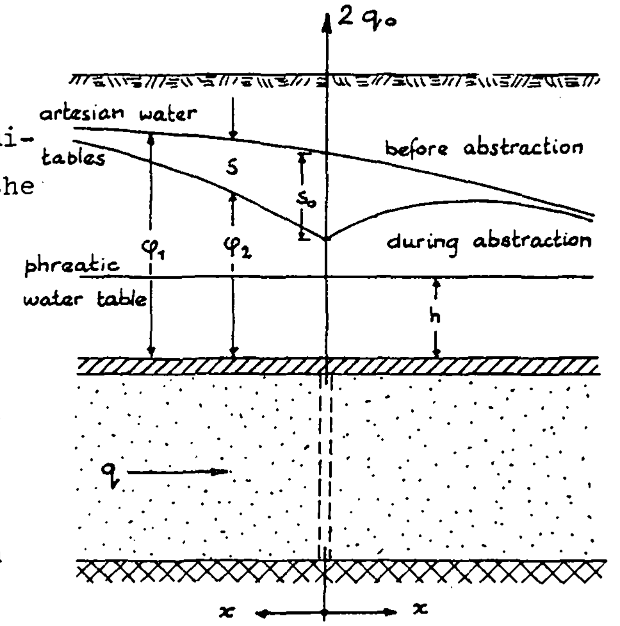
With a constant thickness H, equal to the average value of 22 m, the discharge would have been

$$q_o = kH \frac{h_1 - h_2}{L} = (0.35)10^{-3} (22) \frac{5}{4200} = (9.2)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

which value differs only 3% from the true discharge.

2.03 A leaky artesian aquifer of infinite extent has a constant coefficient of transmissibility kH equal to $(3)10^{-3}$ m^2/sec. The confining layer at the bottom is impervious, the confining layer at the top semi-pervious with a resistance c of $(0.2)10^9$ sec against vertical watermovement. Above the semi-pervious layer an unconfined aquifer with a constant and uniform water table is present.

In the artesian aquifer a fully penetrating gallery is constructed. What is its discharge for a drawdown of 2 m ?



The flow of groundwater in a leaky artesian aquifer may be described with the formulae

$$\phi = Ce^{-x/\lambda} + C'e^{+x/\lambda} + h$$

$$q = C\frac{kH}{\lambda}e^{-x/\lambda} - C'\frac{kH}{\lambda}e^{+x/\lambda}$$

with $\lambda = \sqrt{kHc}$

and C and C' integration constants to be determined from the boundary conditions.

With an aquifer of infinite extent, C' = 0, giving as flow formulae for the aquifer to the left of the gallery

before abstraction $\phi_1 = C_1e^{-x/\lambda} + h \qquad q_1 = C_1\frac{kH}{\lambda}\ e^{-x/\lambda}$

during abstraction $\phi_2 = C_2e^{-x/\lambda} + h \qquad q_2 = C_2\frac{kH}{\lambda}\ e^{-x/\lambda}$

The drawdowns and flowrates due to abstraction thus become

$$s = \phi_1 - \phi_2 = (C_1 - C_2)e^{-x/\lambda}$$

$$q_r = q_1 - q_2 = (C_1 - C_2)\frac{kH}{\lambda}e^{-x/\lambda}$$

combined $q_r = \frac{kH}{\lambda}s$ and at the face of the gallery

$$q_r = \frac{kH}{\lambda}s_o = q_o$$

The same calculation may be made for the aquifer to the right of the gallery, giving as total abstraction

$$2q_o = 2\frac{kH}{\lambda} = 2\sqrt{\frac{kH}{c}}\, s_o$$

With a drawdown s_o of 2 m and the geo-hydrological constants as given, the discharge becomes

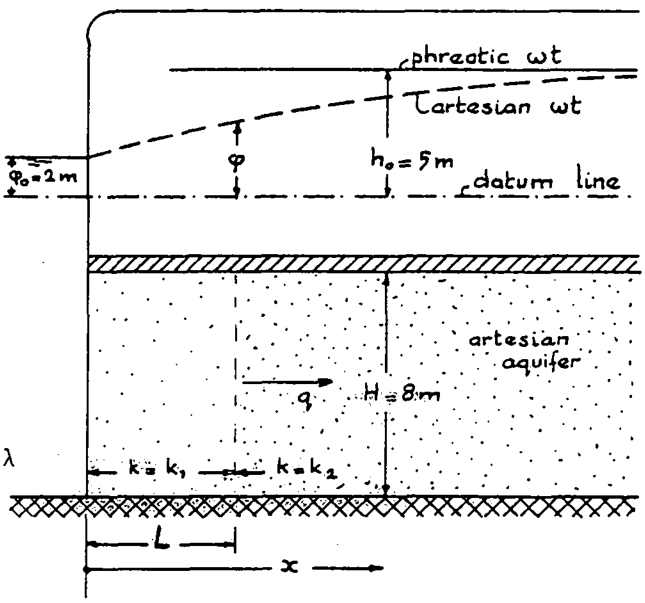
$$2q_o = 2\sqrt{\frac{(3)10^{-3}}{(0.2)10^{9}}}\,(2) = (15.5)10^{-6}\ m^3/m'/sec.$$

2.04 A leaky artesian aquifer is situated above an impervious base and is overlain by a semi-pervious layer with a resistance of $(0.3)10^9$ sec against vertical water movement. Above this layer an unconfined aquifer is present, having a constant and uniform water table at 5 m above datum line. To the right the artesian aquifer extends to infinity, to the left it is bounded by a fully penetrating ditch with a constant water level at 2 m above datum line.

The thickness H of the artesian aquifer is constant at 8 m, but its coefficient of permeability k varies, being equal to $(0.6)10^{-3}$ m/sec for a 500 m wide strip bordering the ditch and equal to $(0.2)10^{-3}$ m/sec at greater distances inland.

What is the outflow of artesian water into the ditch? At what distance from the ditch does the artesian water table reach a level of 4.7 m above datum line?

Flow of groundwater in a leaky artesian aquifer - as shown in the picture at the right - may be described with the formulae



$$\phi = C_1 e^{-x/\lambda} + C_2 e^{+x/\lambda} + h_o$$

$$q = C_1 \frac{kH}{\lambda} e^{-x/\lambda} - C_2 \frac{kH}{\lambda} e^{+x/\lambda}$$

In case under consideration, however, k varies with distance, giving

$0 < x < L$ $\quad k_1 = (0.6)10^{-3}$ m/sec, $\quad k_1H = (4.8)10^{-3}$ m^2/sec

$$\lambda_1 = \sqrt{(4.8)10^{-3}(0.3)10^9} = 1200 \text{ m}$$

$L < x$ $\quad k_2 = (0.2)10^{-3}$ m/sec, $\quad k_2H = (1.6)10^{-3}$ m^2/sec

$$\lambda_2 = \sqrt{(1.6)10^{-3}(0.3)10^9} = 693 \text{ m}$$

The flow equations thus become

$$0 < x < L \qquad \phi_1 = C_1 e^{-x/\lambda_1} + C_2 e^{+x/\lambda_1} + h_o$$

$$q_1 = C_1 \frac{k_1 H}{\lambda_1} e^{-x/\lambda_1} - C_2 \frac{k_1 H}{\lambda_1} e^{+x/\lambda_1}$$

$$L < x \qquad \phi_2 = C_3 e^{-x/\lambda_2} + C_4 e^{+x/\lambda_2} + h_o$$

$$q_2 = C_3 \frac{k_2 H}{\lambda_2} e^{-x/\lambda_2} - C_4 \frac{k_2 H}{\lambda_2} e^{+x/\lambda_2}$$

The integration constants C_1 to C_4 inclusive follow from the boundary conditions

$$x = 0 \qquad \phi_1 = \phi_0 = C_1 + C_2 + h_o$$

$$x = L \qquad \phi_1 = \phi_2 \quad \text{or} \quad C_1 e^{-L/\lambda_1} + C_2 e^{+L/\lambda_1} = C_3 e^{-L/\lambda_2} + C_4 e^{+L/\lambda_2}$$

$$q_1 = q_2 \quad \text{or} \quad C_1 \frac{k_1}{\lambda_1} e^{-L/\lambda_1} - C_2 \frac{k_1}{\lambda_1} e^{+L/\lambda_1} =$$

$$= C_3 \frac{k_2}{\lambda_2} e^{-L/\lambda_2} - C_4 \frac{k_2}{\lambda_2} e^{+L/\lambda_2}$$

$$x \to \infty \qquad \phi_2 = h_o \quad \text{or} \quad C_4 = 0$$

With the data as mentioned above

$$C_1 + C_2 = -3$$

$$C_1 + 2.30\, C_2 = 0.734\, C_3$$

$$C_1 - 2.30\, C_2 = 0.422\, C_3$$

from which follows $C_1 = -2.68$, $C_2 = -0.32$ and $C_3 = -4.64$

The outflow of artesian water into the ditch thus equals

$$- q_{01} = - C_1 \frac{k_1 H}{\lambda_1} + C_2 \frac{k_1 H}{\lambda_2} = (- C_1 + C_2) \frac{k_1 H}{\lambda_1}$$

$$- q_{01} = (2.68 - 0.32) \frac{(4.8)10^{-3}}{1200} = (9.4)10^{-6} \ m^3/m'/sec$$

When it is provisionally assumed that the value $\phi = 4.7$ m is reached at $x > L$, this value follows from

$$4.7 = - 4.64 \ e^{-x/693} + 5.00 \quad \text{or}$$

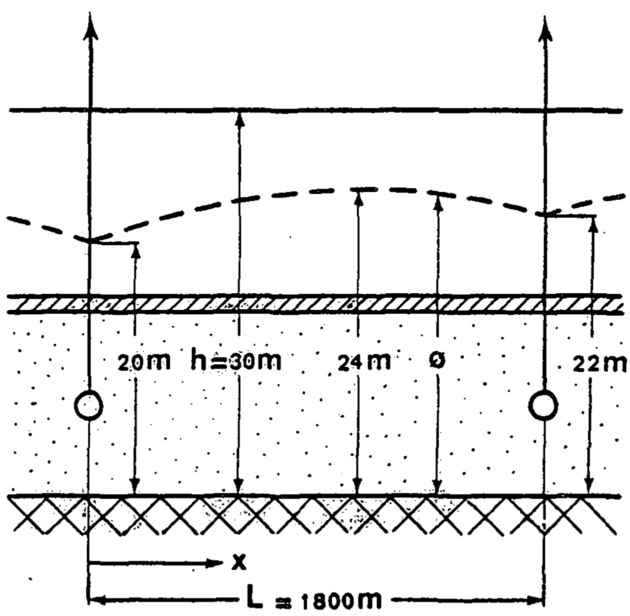
$$e^{-x/693} = \frac{0.30}{4.64} = 0.0646, \quad x = (2.74)(693) = 1900 \ m$$

which value is indeed larger than L = 500 m.

2.05 A leaky artesian aquifer of infinite extent has a thickness H of 15 m, a coefficient of permeability k equal to $(0.4)10^{-3}$ m/sec, is situated above an impervious base and is covered by a semi-pervious layer. Above this semi-pervious layer phreatic water is present with a constant and uniform level rising to 30.0 m above the impervious base.

From the leaky artesian aquifer groundwater is abstracted by means of two parallel ditches with an interval L of 1800 m. The water levels in the two ditches rise to 20.0 and 22.0 m above the impervious base respectively, while between the ditches the maximum artesian water table equals 24.0 m above the impervious base.

What is the resistance c of the semi-pervious layer against vertical water movement?



The one-dimensional flow of groundwater in a leaky artesian aquifer can be described with

$$\phi = C_1 e^{-x/\lambda} + C_2 e^{+x/\lambda} + h \text{ with}$$

$$\lambda = \sqrt{kHc}$$

According to the data provided

$x = 0$, $\phi = 20 = C_1 + C_2 + 30$

$C_1 + C_2 = -10$ (1)

$x = 1800$, $\phi = 22 = C_1 e^{-1800/\lambda} + C_2 e^{+1800/\lambda} + 30$ $\quad C_1 e^{-1800/\lambda} + C_2 e^{+1800/\lambda} = -8$ (2)

At the site of the maximum water level, $\frac{d\phi}{dx} = 0$

$0 = -\frac{C_1}{\lambda} e^{-x/\lambda} + \frac{C_2}{\lambda} e^{+x/\lambda}$ or $e^{x/\lambda} = \sqrt{\frac{C_1}{C_2}}$. Substituted in the formula for ϕ

$$24 = C_1\sqrt{\frac{C_2}{C_1}} + C_2\sqrt{\frac{C_1}{C_2}} + 30 \text{ or } 2\sqrt{C_1 C_2} = -6, C_1 C_2 = 9 \text{ (3)}$$

This gives

$$(1')\quad C_1 + \frac{9}{C_1} = -10,\ C_1^2 + 10C_1 + 9 = 0 \quad C_1 = -\frac{10}{2} \pm \frac{1}{2}\sqrt{100 - 36} = -5 \pm 4$$

$$C_1 = -1,\ C_2 = -9 \qquad \text{and } C_1 = -9,\ C_2 = -1$$

Substituted in (2) with $e^{1800/\lambda} = z$

$$-\frac{1}{z} - 9z = -8 \qquad\qquad -\frac{9}{z} - z = -8$$

$$9z^2 - 8z + 1 = 0 \qquad\qquad z^2 - 8z + 9 = 0$$

$$z = \frac{8}{18} \pm \frac{1}{18}\sqrt{64 - 36} \qquad\qquad z = \frac{8}{2} \pm \frac{1}{2}\sqrt{64 - 36}$$

$$z_1 = 0.738,\ \lambda = -5936 \text{ m} \qquad\qquad z_1' = 6.646,\ \lambda = 950 \text{ m}$$

$$z_2 = 0.150,\ \lambda = -950 \text{ m} \qquad\qquad z_2' = 1.354,\ \lambda = 5936 \text{ m}$$

With $c = \frac{\lambda^2}{kH}$ and $kH = (0.4)10^{-3}(15) = (6)10^{-3}$

$$c = \frac{(950)^2}{(6)10^{-3}} = (0.1505)10^9 \text{ sec} = 4.77 \text{ years}$$

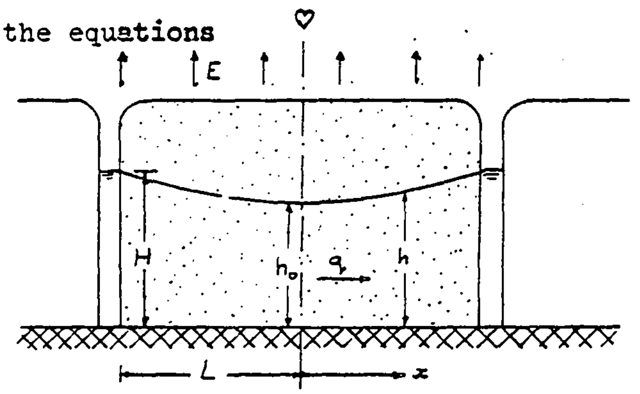
$$c = \frac{(5936)^2}{(6)10^{-3}} = (5.872)10^9 \text{ sec} = 186 \text{ years}$$

With a resistance of 186 years, the recharge of the leaky artesian aquifer from above would be extremely small and the possibilities of groundwater recovery next to negligible. The correct value consequently is $c = 4.77$ years.

2.11 Two parallel fully penetrating ditches have equal waterlevels of 5 m above the impervious base. The unconfined aquifer in -between has a width of 300 m and a coefficient of permeability k equal to $(0.25)10^{-3}$ m/sec. Due to evapo-transpiration the aquifer loses water in an amount of $(0.12)10^{-6}$ m/sec.

What is the lowest level of the ground-water table?

With the notations as indicated in the figure at the right, the equations of flow become



Darcy $q = - kh \frac{dh}{dx}$

continuity $q = - Ex$

combined $hdh = \frac{E}{k} xdx$

integrated between the limits $x = 0$, $h = h_o$ and $x = L$, $h = H$

$$H^2 - h_o^2 = \frac{E}{k} L^2$$

With the data supplied

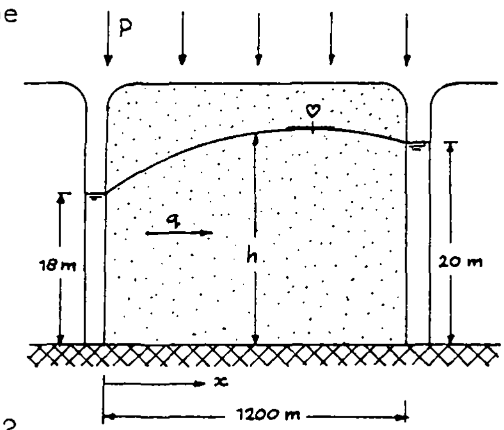
$$25 - h_o^2 = \frac{(0.12)10^{-6}}{(0.25)10^{-3}} (150)^2$$

$$h_o^2 = 25 - 10.8 = 14.2, \quad h_o = 3.77 \text{ m.}$$

2.12 An unconfined aquifer is situated above an impervious base and is composed of sand with a coefficient of permeability k equal to $(0.15)10^{-3}$ m/sec. In this aquifer two fully penetrating ditches form a strip of land with a constant width of 1200 m. The water levels in the ditches rise to 18 and 20 m above the impervious base respectively, while the recharge by rainfall minus evapo-transpiration losses P amounts to $(23)10^{-9}$ m/sec.

What is the outflow of groundwater to both ditches and what is the maximum elevation of the groundwater table?

Going out from the picture at the right, the equations of flow may be written as



Darcy $q = - kh \frac{dh}{dx}$

continuity $\frac{dq}{dx} = P$ or $q = Px+C_1$

combined $hdh = - \frac{P}{k} xdx - \frac{C_1}{k} dx$

integrated $h^2 = - \frac{P}{k} x^2 - \frac{2C_1}{k} x + C_2$

With the values of P and k as mentioned above, the boundary conditions give

$$x = 0,\ h = 18;\quad (18)^2 = C_2 \quad \text{or} \quad C_2 = 324$$

$$x = 1200,\ h = 20;$$

$$(20)^2 = - \frac{(23)10^{-9}}{(0.15)10^{-3}} (1200)^2 - \frac{2\ C_1}{(0.15)10^{-3}} (1200) + 324$$

from which follows

$$C_1 = - (18.7)10^{-6}$$

$$q = (23)10^{-9}\ x\ -(18.7)10^{-6}$$

The outflow of groundwater in the ditches occurs at

$$x = 0, \quad - q = (18.7)10^{-6}\ m^3/m'/sec$$

$$x = 1200, \quad q = (23)10^{-9}(1200) - (18.7)10^{-6} \quad \text{or}$$

$$q = (27.6)10^{-6} - (18.7)10^{-6} = (8.9)10^{-6}\ m^3/m'/sec$$

Inside the strip of land the groundwater table reaches its maximum elevation at the water divide. Here $q = 0$ or

$$0 = (23)10^{-9}\ x - (18.7)10^{-6} \quad , \ x = 812\ m$$

This gives as water table elevation

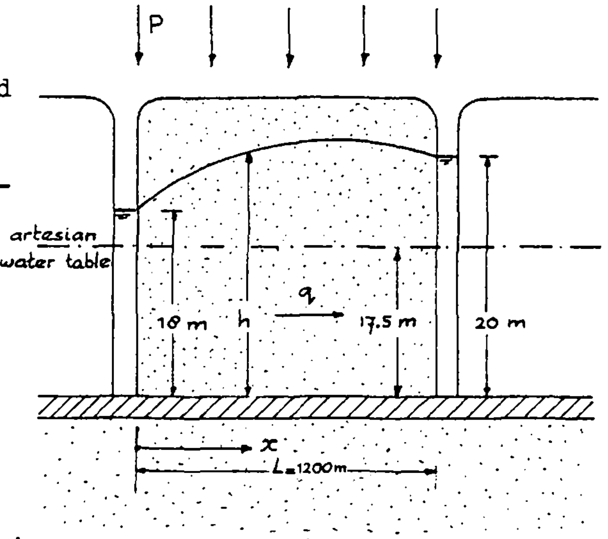
$$h^2 = - \frac{(23)10^{-9}}{(0.15)10^{-3}} (812)^2 + \frac{(2)(18.7)10^{-6}}{(0.15)10^{-3}} (812) + 324$$

$$h^2 = - 101 + 203 + 324 = 426, \quad h = 20.7\ m$$

2.13 An unconfined aquifer is situated above a semi-pervious layer and is composed of sand with a coefficient of permeability k equal to $(0.15)10^{-3}$ m/sec. In this aquifer two fully penetrating ditches form a strip of land with a constant width of 1200 m. The water levels in the ditches rise to 18 and 20 m above the semi-pervious layer respectively, while the recharge by rainfall minus the losses due to evapotranspiration P amounts to $(23)10^{-9}$ m/sec.

The resistance c of the semi-pervious layer against vertical water movement amounts to $(320)10^{6}$ sec. Below this layer artesian water is present with a constant and uniform level ϕ rising to 17.5 m above the base of the unconfined aquifer.

What is the outflow of groundwater to both ditches and what is the maximum elevation of the groundwater table?



When for the unconfined aquifer above the semi-pervious layer a constant coefficient of transmissibility kH may be assumed, the groundwater flow in this aquifer is governed by

$$h = C_1 e^{-x/\lambda} + C_2 e^{+x/\lambda} + Pc + \phi$$

$$q = C_1 \frac{kH}{\lambda} e^{-x/\lambda} - C_2 \frac{kH}{\lambda} e^{+x/\lambda} \quad \text{with}$$

$\lambda = \sqrt{kHc}$ and the integration constants C_1 and C_2 to be determined from the boundary conditions.

When the average saturated thickness of the aquifer is provisionally estimated at 19 m, the geo-hydrologic constants become

$$kH = (0.15)10^{-3}\ (19) = (2.85)10^{-3}\ m^2/sec$$

$$\lambda = \sqrt{(2.85)10^{-3}\ (320)10^{6}} = 955\ m$$

$$Pc = (23)10^{-9}(320)10^{6} = 7.36\ m$$

Substitution of the boundary conditions gives

$x = 0 \quad , h = 18 = C_1 + C_2 + 7.36 + 17.5$

$x = 1200, h = 20 = C_1 e^{-1200/955} + C_2 e^{+1200/955} + 7.36 + 17.5$ Simplified

$$-6.86 = C_1 + C_2$$

$$-4.86 = \frac{C_1}{3.51} + (3.51)C_2 \quad \text{from which follows}$$

$$C_1 = -5.96 \qquad C_2 = -0.90$$

This gives as rate of flow

$$q = -(5.96)\frac{(2.85)10^{-3}}{955} e^{-x/955} + (0.90)\frac{(2.85)10^{-3}}{955} e^{+x/955}$$

$$q = -(17.8)10^{-6} e^{-x/955} + (2.7)10^{-6} e^{+x/955}$$

The outflow of groundwater in both ditches thus become

$x = 0, \quad -q = +(17.8)10^{-6} -(2.7)10^{-6} = (15.1)10^{-6} \ m^3/m'/sec$

$x = 1200, \ q = -(17.8)10^{-6} e^{-1200/955} + (2.7)10^{-6} e^{+1200/955} =$

$$= (4.4)10^{-6} \ m^3/m'/sec$$

Inside the strip of land the groundwater table reaches its maximum elevation at the water divide. Here $q = 0$ or

$$0 = -(17.8)10^{-6} e^{-x/955} + (2.7)10^{-6} e^{+x/955}$$

$$e^{2x/955} = \frac{17.8}{2.7} = 6.60 = e^{1.89} \quad \text{or } x = \frac{(1.89)(955)}{2} = 902 \text{ m}$$

and $\quad h = -5.96 \ e^{-902/955} - 0.90 \ e^{+902/955} + 7.36 + 17.5$

$$h = -2.32 - 2.31 + 7.36 + 17.5 = 20.2 \text{ m}$$

The calculations made above in the meanwhile, are based on the assumption that the average saturated thickness H of the unconfined aquifer amounts to 19 m. To check this assumption, the loss of water downward through the semi-pervious layer will be calculated. On one hand this loss equals the recharge by rainfall minus the outflows to the sides

$$q_i = (23)10^{-9}\ (1200) - (15.1)10^{-6} - (4.4)10^{-6} = (8.1)10^{-6}$$

With Δ as difference in water level above and below the semi-pervious layer, this loss on the other hand equals

$$q_i = \frac{\Delta}{c} L = \frac{\Delta}{(320)10^6}\ (1200) = \frac{\Delta}{(0.267)10^6}$$

Equality of both losses gives

$$\Delta = (8.1)10^{-6}(0.267)10^6 = 2.16 \text{ m}$$

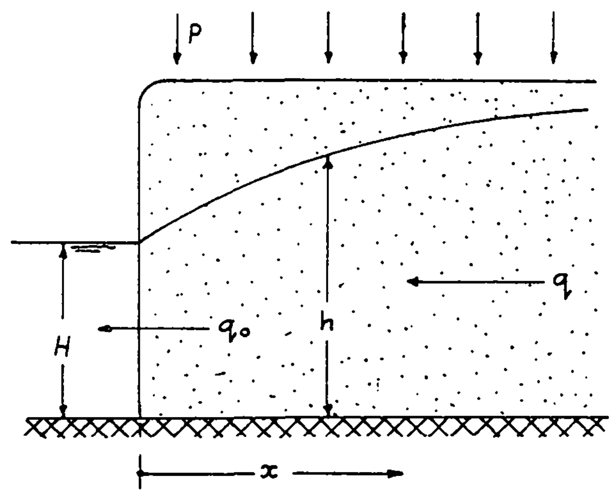
With the artesian water table at 17.5 m, the average water table depth in the unconfined aquifer above the semi-pervious layer equals

$$H = 17.5 + 2.16 = 19.7 \text{ m}$$

or 4% more as the assumed value of 19 m. This difference, however, is still so small that recalculation is hardly required.

2.14 A semi-infinite unconfined aquifer is situated above a horizontal impervious base and bounded by a fully penetrating ditch. The water level in the ditch is constant at 20.0 m above the base, while the groundwater table rises to 22.0 and 29.0 m above the base at distances of 500 and 3000 m from the ditch respectively. The outflow of groundwater into the ditch equals $(24)10^{-6}$ m^3/m'/sec.

How large is the average groundwater recharge P by rainfall? What is the value of the coefficient of permeability k of the aquifer?



With the notations of the figure at the right, the equations of flow may be written as

Darcy $\quad q = kh\dfrac{dh}{dx}$

continuity $q = q_o - Px$

combined $hdh = \dfrac{q_o}{k}dx - \dfrac{P}{k}xdx$

integrated $h^2 = \dfrac{2q_o}{k}x - \dfrac{P}{k}x^2 + C$ With the boundary conditions x = 0,

$h = H$, $H^2 = C$ or

$$h^2 = H^2 + \frac{2q_o}{k}x - \frac{P}{k}x^2$$

the water table elevations at x = 500 m and x = 3000 m thus become

$$(22)^2 = (20)^2 + \frac{(48)10^{-6}}{k}(500) - \frac{P}{k}(500)^2$$

$$\text{or } P = (96)10^{-9} - (336)10^{-6}k$$

$$(29)^2 = (20)^2 + \frac{(48)10^{-6}}{k}(3000) - \frac{P}{k}(3000)^2$$

$$P = (16)10^{-9} - (49)10^{-6}k$$

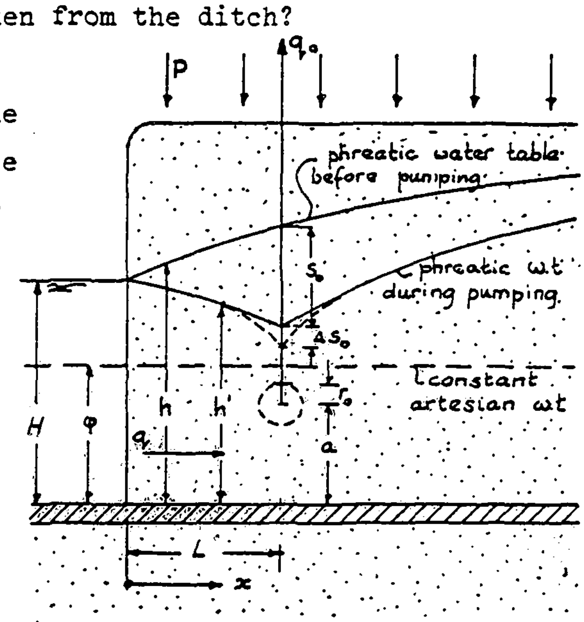
from which equations follows

$$k = (0.28)10^{-3} \text{ m/sec}, \quad P = (2.3)10^{-9} \text{ m/sec}$$

2.15 A semi-infinite unconfined aquifer is situated above a semi-pervious layer and bounded by a fully penetrating ditch with a constant and uniform water level at 25 m above the top of the semi-pervious layer. The unconfined aquifer has a coefficient of permeability k equal to $(0.16)10^{-3}$ m/sec and is recharged by rainfall P in an amount of $(50)10^{-9}$ m/sec. The semi-pervious layer has a resistance c of $(80)10^{6}$ sec against vertical water movement. Below this semi-pervious layer artesian water is present. The artesian water table is uniform and constant at 23.5 m above the top of the semi-pervious layer.

At a distance of 300 m parallel to the ditch a circular gallery with an outside diameter of 0.4 m is constructed, with its centre 12 m above the top of the semi-pervious layer. From this gallery groundwater is abstracted in a constant amount of $(65)10^{-6}$ m^3/m'/sec.

What is the drawdown at the face of the gallery and what is the remaining water table depth at 1 km from the ditch? Which part of the groundwater abstraction is taken from the ditch?



Before pumping the gallery, the water table elevation h and the rate of flow q in the unconfined aquifer are given by

$$h = C_1 e^{-x/\lambda} + C_2 e^{+x/\lambda} + Pc + \phi$$

$$q = C_1 \frac{kH}{\lambda} e^{-x/\lambda} - C_2 \frac{kH}{\lambda} e^{+x/\lambda} \quad \text{with}$$

$$\lambda = \sqrt{kHc}$$

The values of the integration constant C_1 and C_2 follow from the boundary conditions

at $x = 0$, $h = H = C_1 + C_2 + Pc + \phi$

at x to infinity, h remains finite or $C_2 = 0$

With the data as given

$$25 = C_1 + (50)10^{-9}\,(80)10^{6} + 23.5 = C_1 + 27.5, \quad C_1 = -2.50$$

$$\lambda = \sqrt{(0.16)10^{-3}(25)(80)10^{6}} = 565 \text{ m} \qquad \text{and}$$

$$h = -2.50\ e^{-x/565} + 27.5$$ This gives at the location of the gallery $x = L = 300$ m as water table depth before pumping

$$h_o = -2.5\ e^{-300/565} + 27.5 = -1.47 + 27.5 = 26.0 \text{ m}$$

During pumping, the aquifer may be subdivided in 2 parts

$0 < x < L$

$$h' = C_1' e^{-x/\lambda} + C_2' e^{+x/\lambda} + Pc + \phi$$

$$q' = C_1' \frac{kH}{\lambda} e^{-x/\lambda} - C_2' \frac{kH}{\lambda} e^{+x/\lambda}$$ with as boundary conditions

$$x = 0, \quad h' = H = C_1' + C_2' + Pc + \phi$$

$L < x < \infty$

$$h'' = C_1'' e^{-x/\lambda} + C_2'' e^{+x/\lambda} + Pc + \phi$$

$$q'' = C_1'' \frac{kH}{\lambda} e^{-x/\lambda} - C_2'' \frac{kH}{\lambda} e^{+x/\lambda}$$ with as boundary conditions

$x \to \infty$, h'' remains finite or $C_2'' = 0$

At $x = L$ the boundary conditions are

$$h' = h'' \qquad C_1' e^{-L/\lambda} + C_2' e^{+L/\lambda} + Pc + \phi = C_1'' e^{-L/\lambda} + Pc + \phi$$

$$q' - q'' = q_o \qquad C_1' \frac{kH}{\lambda} e^{-L/\lambda} - C_2' \frac{kH}{\lambda} e^{+L/\lambda} - C_1'' \frac{kH}{\lambda} e^{-L/\lambda} = q_o$$ With

$$e^{+L/\lambda} = e^{+300/565} = 1.70, \quad \frac{kH}{\lambda} = \frac{(0.16)(10^{-3})(25)}{565} = (7.08)10^{-6}$$

$$25 = C_1' + C_2' + 27.5 \qquad \text{or} \qquad C_1' + C_2' = -2.5$$

$$\frac{C_1'}{1.70} + 1.70\ C_2' = \frac{C_1''}{1.70} \qquad C_1' + 2.89\ C_2' - C_1'' = 0$$

$$\frac{C_1'}{1.70} - 1.70\ C_2' - \frac{C_1''}{1.70} = \frac{(65)10^{-6}}{(7.08)10^{-6}} \qquad C_1' - 2.89\ C_2' - C_1'' = 15.6$$

from which equations follows

$$C_1' = +0.20 \qquad C_2' = -2.70 \qquad C_1'' = -7.61$$

At the face of the gallery, x = L = 300 m, the water table elevation thus becomes

$$h_o' = \frac{0.20}{1.70} - (2.70)(1.70) + 27.5 = 0.12 - 4.59 + 27.5 = 23.0 \text{ m}$$

giving as drawdown of the fully penetrating gallery

$$s_o = h_o - h_o' = 26.0 - 23.0 = 3.0 \text{ m}$$

To this drawdown must be added the influence of partial penetration. With the gallery about halfway the saturated depth of the aquifer

$$\Delta s_o = \frac{q_o}{2\pi k} \ln \frac{H}{2\pi r_o}$$

$$\Delta s_o = \frac{(65)10^{-6}}{2\pi(0.16)10^{-3}} \ln \frac{25}{2\pi(0.2)} = 0.065 \ln 20 = 0.2 \text{ m, together}$$

$$s_o' = s_o + \Delta s_o = 3.0 + 0.2 = 3.2 \text{ m}$$

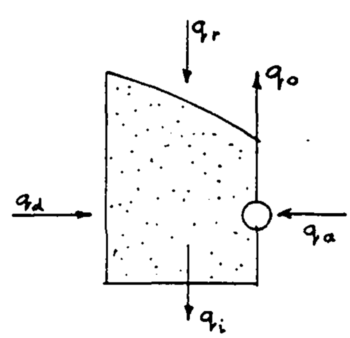
At x = 1 km = 1000 m, the water table depth during pumping equals $h'' = C_1'' e^{-x/\lambda} + Pc + \phi$

$$h'' = -7.61\, e^{-1000/565} + 27.5 = -1.3 + 27.5 = 26.2 \text{ m}$$

To determine which part of the gallery yield is taken from the bounding ditch, the water balance for the strip of land between the gallery and the ditch must be determined. The flows as indicated in the picture at the right equal

$$q_r = PL = (50)10^{-9}(300) =$$

$$= (15)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$



$$q_d = C_1' \frac{kH}{\lambda} - C_2' \frac{kH}{\lambda}$$

$$q_d = (0.20)(7.08)10^{-6} + (2.70)(7.08)10^{-6} = (21)10^{-6}$$

$$-q_a = C_1'' \frac{kH}{\lambda} e^{-L/\lambda}$$

$$q_a = (7.61)(7.08)10^{-6} \frac{1}{1.70} = (32)10^{-6}$$

$$q_o = (65)10^{-6}$$

$$q_i = q_r + q_d + q_a - q_o = (15)10^{-6} + (21)10^{-6} + (32)10^{-6} -$$

$$-(65)10^{-6} = (3)10^{-6}$$

From the inflow q_d from the ditch at $(21)10^{-6}$ $m^3/m'/sec$, an amount of $(3)10^{-6}$ will percolate downward through the semi-pervious layer to the artesian aquifer below, leaving $(18)10^{-6}$ to be abstracted by the gallery. The yield of the gallery thus consists for

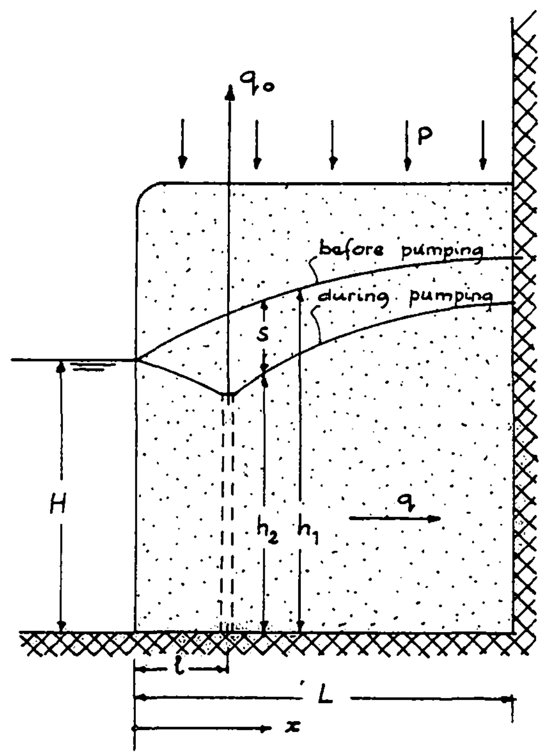
$$\frac{(18)10^{-6}}{(65)10^{-6}} 100 \text{ or } 28\% \text{ of water derived from the ditch.}$$

2.16 A strip of land with a constant width of 2000 m is situated between a fully penetrating ditch and an outcropping of impervious rocks. The geo-hydrological profile of this strip shows an unconfined aquifer above an impervious base with a coefficient of permeability k equal to $(0.24)10^{-3}$ m/sec, recharged by rainfall in an amount of $(18)10^{-9}$ m/sec. The water level in the bounding ditch is constant at 20 m above the impervious base.

At a distance of 500 m parallel to the ditch a fully penetrating gallery is constructed and pumped at a constant rate of $(30)10^{-6}$ $m^3/m'/sec$.

What is the drawdown of the water table at the gallery and at the rock boundary?

Before pumping the gallery, the flow in the strip of land is governed by



Darcy $q = -kh_1 \frac{dh_1}{dx}$

continuity $q = -P(L - x)$

combined $h_1 dh_1 = \frac{PL}{k}dx - \frac{Px}{k}dx$

integrated $h_1^2 = \frac{2PL}{k}x - \frac{P}{k}x^2 + C$

With the boundary condition $x = 0$, $h_1 = H$, $C = H^2$, substituted

$$h_1^2 - H^2 = \frac{2PL}{k}x - \frac{P}{k}x^2$$

In the case under consideration

$$h_1^2 - (20)^2 = \frac{(2)(18)10^{-9}(2000)}{(0.24)10^{-3}}x - \frac{(18)10^{-9}}{(0.24)10^{-3}}x^2 \quad \text{or}$$

$$h_1^2 = 400 + \frac{x}{3.33} - (\frac{x}{115})^2$$

$x = l = 500\text{m}, \; h_1^2 = 400 + \frac{500}{3.33} - \left(\frac{500}{115}\right)^2 = 400+150-19 = 531, \; h_{1,l} = 23.1 \text{ m}$

$x = L = 2000\text{m}, \; h_1^2 = 400 + \frac{2000}{3.33} - \left(\frac{2000}{115}\right)^2 = 400+600-303 = 697, \; h_{1,L} = 26.4 \text{ m}$

During abstraction the equations of flow are

$$0 < x < l$$

Darcy $\quad q = -kh_2 \frac{dh_2}{dx}$

continuity $\quad q = q_o - P(L - x)$

combined $\quad h_2 dh_2 = -\frac{q_o}{k} dx + \frac{PL}{k} dx - \frac{Px}{k} dx$

intergrated between the limits $x = 0$, $h_2 = H$ and $x = l$, $h_2 = h_{2,l}$

$$h_{2,l}^2 - H^2 = -\frac{2q_o}{k} l + \frac{2PL}{k} l - \frac{P}{k} l^2$$

$$h_{2,l}^2 - (20)^2 = -\frac{(2)(30)10^{-6}}{(0.24)10^{-3}} (500) + \frac{500}{3.33} - \left(\frac{500}{115}\right)^2$$

$$h_{2,l}^2 = 400 - 125 + 150 - 19 = 406, \; h_o = 20.1 \text{ m}$$

The drawdown at the face of the gallery thus becomes

$$s_l = h_{1,l} - h_{2,l} = 23.1 - 20.1 = 3.0 \text{ m}$$

$$l < x < L$$

Darcy $\quad q = -kh_2 \frac{dh_2}{dx}$

continuity $\quad q = -P(L - x)$

combined $\quad h_2 dh_2 = \frac{PL}{k} dx - \frac{Px}{k} dx$

integrated between the limits $x = l$, $h = h_{2,l}$ and $x = L$, $h_{2,L}$

$$h^2_{2,L} - h^2_{2,1} = \frac{2PL}{k}(L - 1) - \frac{P}{k}(L^2 - 1^2)$$

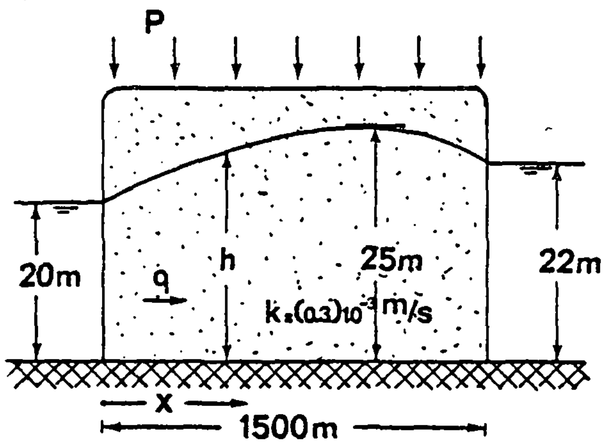
$$h^2_{2,L} - 406 = \frac{1500}{3.33} - (\frac{2000}{115})^2 + (\frac{500}{115})^2$$

$$h^2_{2L} = 406 + 450 - 303 + 19 = 572, \quad h_{2L} = 23.9 \text{ m}$$ and

$$s_L = h_{1,L} - h_{2,L} = 26.4 - 23.9 = 2.5 \text{ m}$$

2.17 An unconfined aquifer is situated above an impervious base and is composed of sand with a coefficient of permeability k equal to $(0.3)10^{-3}$ m/sec. In this aquifer two fully penetrating ditches form a strip of land with a constant width of 1500 m. The water level in the left-hand ditch rises to 20 m above the impervious base, the water level in the right-hand ditch to 22 m, while the maximum groundwater table elevation inside the strip equals 25 m above the base.

What is the value of the recharge P by rainfall?



With the notations as indicated in the figure on the right, the equations of flow may be written as

Darcy $\quad q = -kh \dfrac{dh}{dx}$

Continuity $\quad \dfrac{dq}{dx} = P$ or $q = Px + C_1$

combined $\quad hdh = -\dfrac{P}{k} x\, dx - \dfrac{C_1}{k} dx$

integrated $\quad h^2 = -\dfrac{P}{k} x^2 - \dfrac{2C_1}{k} \cdot x + C_2$

Substitution of the boundary conditions gives with the known value of k

$x = 0$ m, $h = 20$ m $\qquad 400 = C_2$

$x = 1500$ m, $h = 22$ m $\qquad 484 = -(7.5)10^9\, P - (10)10^6\, C_1 + C_2$

Inside the strip of land, the highest groundwater level occurs at the water divide. Here $q = 0$ or $x = -\dfrac{C_1}{P}$

$x = -\dfrac{C_1}{P}$, $h = 25$ m $\qquad 625 = -\dfrac{C_1^2}{(0.3)10^{-3}P} + \dfrac{2C_1^2}{(0.3)10^{-3}P} + C_2$

Elimination of $C_2 = 400$ gives

$$84 = -(7.5)10^9 P - (10)10^6 C_1$$

$$225 = \frac{C_1^2}{(0.3)10^{-3}P}\ ,\ C_1^2 = (67.5)10^{-3}P,\ C_1 = -\ 0.2598\sqrt{P}$$

$$P - (0.3464)10^{-3}\sqrt{P} + (11.2)10^{-9} = 0$$

$$\sqrt{P} = +\frac{(0.3464)10^{-3}}{2} \pm \frac{1}{2}\sqrt{(0.3464)^2 10^{-6} - (4)(11.2)10^{-9}}$$

$$\sqrt{P} = +\ (0.1732)10^{-3} + (0.1371)10^{-3} = (0{,}3103)10^{-3}$$

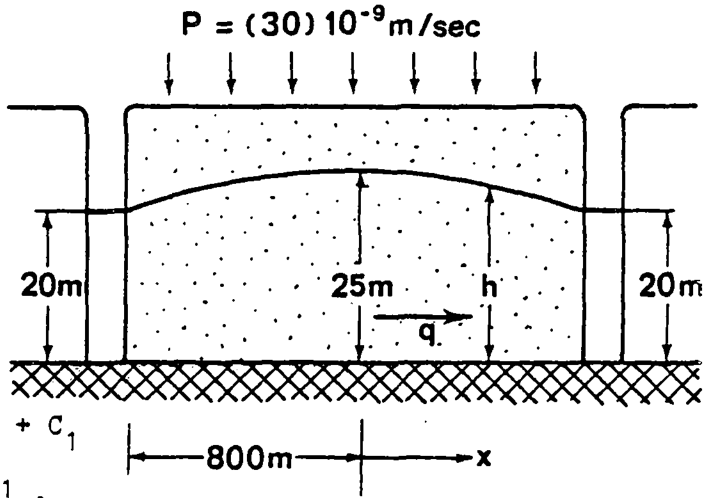
$$P = (96.3)10^{-9}\ \text{m/sec} = 3.04\ \text{m/jaar}$$

2.18 An unconfined aquifer is situated above a horizontal impervious base and is recharged by residual rainfall P in an amount of $(30)10^{-9}$ m/sec. In this aquifer two fully penetrating ditches form a strip of land with a constant width of 1600 m. The waterlevels in both ditches are the same, rising to 20 m above the impervious base, while in the centre of the strip of land the water table attains a height of 25 m above the base.

Questions:

a. What is the value of the coefficient of permeability?

b. What is the maximum lowering of the groundwater table when in the centre of the strip of land a fully penetrating gallery is pumped at a constant rate of $(24)10^{-6}$ m^3/m^1,sec?



With the notations as indicated in the picture on the right, the equations of flow become

Darcy $q = -kh\frac{dh}{dx}$

continuity $\frac{dq}{dx} = P$ or $q = Px + C_1$

combined $hdh = -\frac{P}{k}xdx - \frac{C_1}{k}dx$

integrated $h^2 = -\frac{P}{k}x^2 - \frac{2C_1}{k} + C_2$

Substitution of the boundary conditions gives

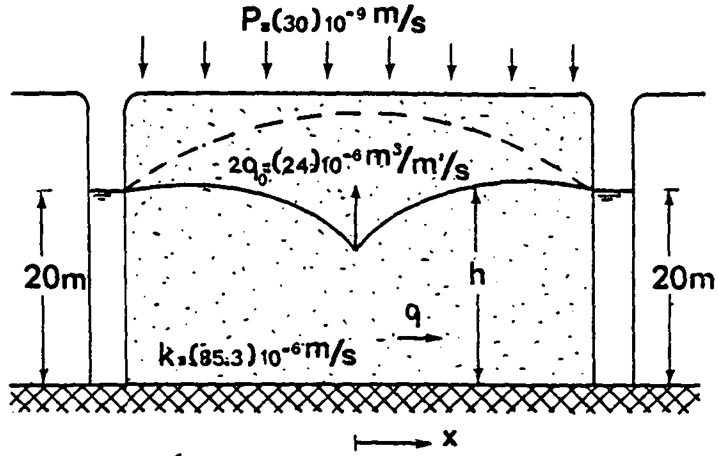
$x = 0$, $h = 25$ m $\quad 625 = C_2$

$x = 800$, $h = 20$ m $\quad 400 = -\frac{(19.2)10^{-3}}{k} - \frac{1600\ C_1}{k} + C_2$

$x = 0$, $q = 0$ $\quad 0 = C_1$

or after elimination of C_1 and C_2

$$400 = -\frac{(19.2)10^{-3}}{k} + 625, \quad k = \frac{(19.2)10^{-3}}{225} = (85.3)10^{-6} \text{ m/sec}$$



With groundwater abstraction by a fully penetrating gallery in the centre, the equation of flow remains the same and only the boundary conditions change

$x = 0 \qquad q = -(12)10^{-6} = C_1$

$x = 800$ m, $H = 20$ m, $400 = -225 + (18.76)10^{6}C_1 + C_2$

This gives $\quad C_1 = -(12)10^{-6},\ C_2 = 400$

The maximum lowering of the groundwater table occurs in the centre, at $x = 0$

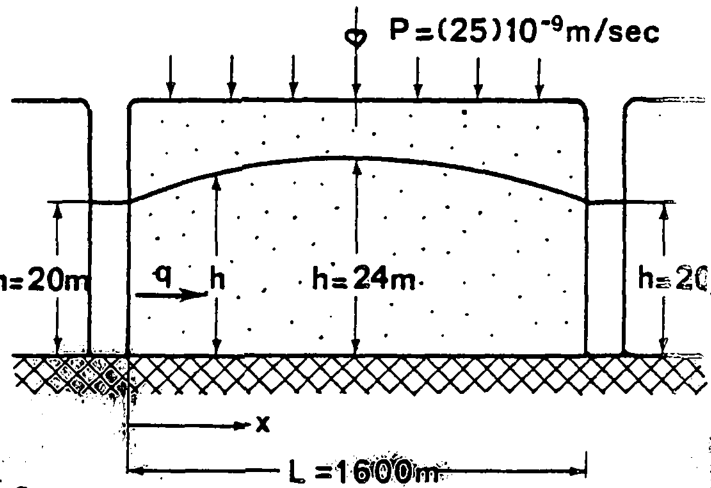
$$h^2 = C_2 = 400 \quad , h = 20.0 \text{ m} \quad , s_o = 5 \text{ m}$$

2.19 An unconfined aquifer is situated above an impervious base and has a recharge by residual rainfall P equal to $(25)10^{-9}$ m/sec. In this aquifer two parallel and fully penetrating ditches form a strip of land with a constant width L of 1600 m. The water levels in both ditches are the same at 20.0 m above the impervious base, while in the centre of the strip of land the groundwater table rises to 24.0 m above the base.

It is planned to construct a fully penetrating gallery at a distance l = 200 m parallel to the left-hand ditch and to abstract from this gallery groundwater in an amount of $(0.08)10^{-3}$ m^3/m,sec.

What will be the remaining water table depth at the gallery and how much water from the bounding ditches will be induced to enter the aquifer?

To determine the coefficient of permeability of the aquifer, the existing situation must first be analysed. With the notations of the picture at the right, the equations of flow may be written as



Darcy $\quad q = -kh \frac{dh}{dx}$

continuity $\quad \frac{dq}{dx} = P$ or $q = Px + C_1$

combined $\quad hdh = -\frac{P}{k} xdx - \frac{C_1}{k} dx$

integrated $\quad h^2 = -\frac{P}{k} x^2 - \frac{2C_1}{k} x + C_2$

Substitution of the boundary conditions gives

x = 0 , h = 20 m $\quad 400 = C_2$

x = 800 m, h = 24 m $\quad 576 = -\frac{(25)10^{-9}}{k}(800)^2 - \frac{2C_1}{k}(800) + C_2$

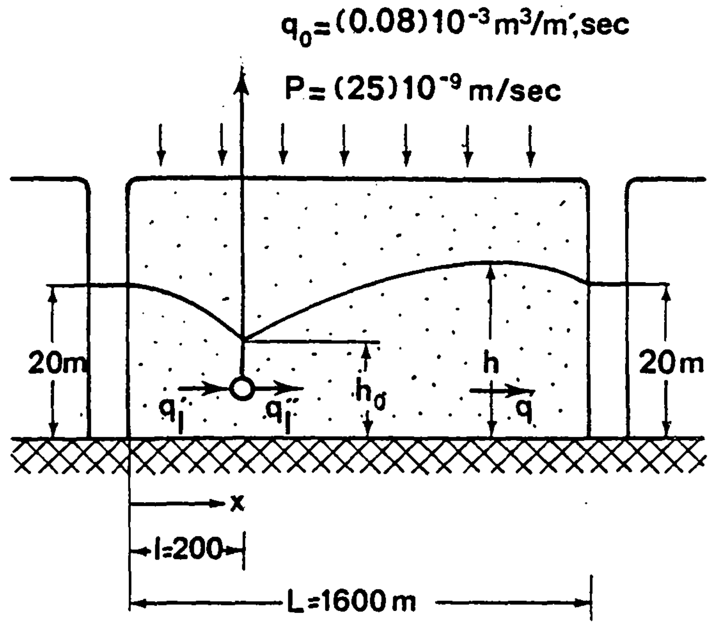
x = 800 m, q = 0 $\quad 0 = (25)10^{-9}(800) + C_1$

from which follows $\quad C_1 = -(20)10^{-6}$, $C_2 = 400$ and

$$576 = -\frac{(16)10^{-3}}{k} + \frac{(32)10^{-3}}{k} + 400$$

$$\frac{(16)10^{-3}}{k} = 176, \quad k = \frac{(16)10^{-3}}{176} = (91)10^{-6} \text{ m/sec}$$

With abstraction, the equation of flow are exactly the same. With other boundary conditions, however, the integration C_1 and C_2 will also have other values.
This gives



$0 < x < l$

$$q = Px + C_1'$$
$$h^2 = -\frac{P}{k}x^2 - \frac{2C_1'}{k}x + C_2'$$

and with the boundary conditions

$x = 0$, $h = 20$ $\qquad 400 = C_2'$

$x = 200$, $h = h_o$ $\qquad h_o^2 = -\frac{(25)10^{-9}}{(91)10^{-6}}(200)^2 - \frac{2C_1'}{(91)10^{-6}}(200) + C_2'$

$x = 200$, $q = q_1'$ $\qquad q_1' = (25)10^{-9}(200) + C_1'$ or

$$h_o^2 = 389.01 - (4.396)10^6\ C_1'$$

$$q_1' = (5)10^{-6} + C_1'$$

$l < x < L$

$$q = Px + C_1''$$
$$h^2 = -\frac{P}{k}x^2 - \frac{2C_1''}{k}x + C_2''$$

and with the boundary conditions

$x = 200$, $h = h_o$ $\qquad h_o^2 = -\frac{(25)10^{-9}}{(91)10^{-6}}(200)^2 - \frac{2C_1''}{(91)10^{-6}}(200) + C_2''$

$x = 1600$, $h = 20$ $\qquad 400 = -\frac{(25)10^{-9}}{(91)10^{-6}}(1600)^2 - \frac{2C_1''}{(91)10^{-6}}(1600) + C_2''$

$x = 200$, $q = q_1''$ $\qquad q_1'' = (25)10^{-9}(200) + C_1''$ or

$$h_o^2 = 1092.31 + (30.769)10^6 C_1''$$

$$q_1'' = (5)10^{-6} + C_1''$$

For $x = 1$, both values of h_o^2 must be the same

$$389.01 - (4.396)10^6 C_1' = 1092.31 + (30.769)10^6 C_1''$$

while the abstraction equals

$$q_o = (0.08)10^{-3} = q_1' - q_1'' = (5)10^{-6} + C_1' - (5)10^{-6} - C_1'' \quad \text{or}$$

$$C_1' + 7C_1'' = -(160)10^{-6}$$

$$C_1' - C_1'' = (0.08)10^{-3}$$

from which follows

$$C_1' = (50)10^{-6}$$

$$C_1'' = -(30)10^{-6}$$

and $$h_o^2 = 389.01 - (4.396)10^6(50)10^{-6} = 169.21, \quad h_o = 13.00 \text{ m}$$

The inflow from the left-hand ditch equals

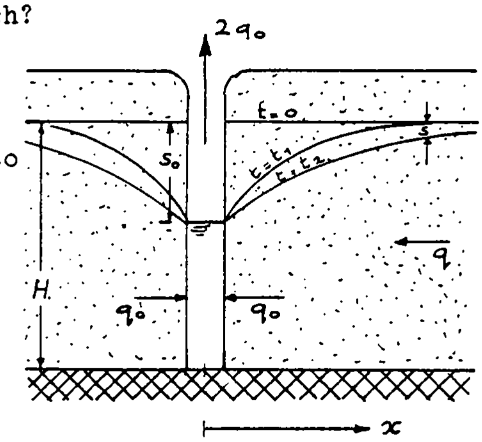
$$x = 0 \qquad q = Px + C_1' = C_1' = (50)10^{-6} \text{ m}^3\text{/m,sec}$$

and the outflow into the right-hand ditch

$$x = 1600 \quad q = Px + C_1'' = (25)10^{-9}(1600) - (30)10^{-6} = (10)10^{-6} \text{ m}^3\text{/m',sec.}$$

2.21 An unconfined aquifer of infinite extend is situated above a horizontal impervious base. The coefficient of transmissibility kH of this aquifer amounts to $(9)10^{-3} m^2/sec$, its specific yield μ to 20%.

In this aquifer a fully penetrating ditch is constructed. At t = 0 the water level in this ditch is lowered suddenly by 3.5 m. How much groundwater flows into this ditch during a 2 month period? What is at the end of this period the rate of flow and the drawdown in a point 500 m from the ditch?



The outflow of groundwater into the ditch equals

$$2q_o = \frac{2s_o}{\sqrt{\pi}} \sqrt{\mu kH} \frac{1}{\sqrt{t}}$$

Over a period T it sums up to

$$\Sigma 2q_o = \int_0^T \frac{2s_o}{\sqrt{\pi}} \sqrt{\mu kH} \frac{1}{\sqrt{t}} dt = \frac{4s_o}{\sqrt{\pi}} \sqrt{\mu kH} \sqrt{T}$$

With T = 2 month = 61 days = $(5.26)10^6$ sec

$$\Sigma 2q_o = \frac{(4)(3.5)}{\sqrt{\pi}} \sqrt{(0.20)(9)10^{-3}} \sqrt{(5.26)10^6} = 770 \ m^3/m'$$

At a point x = 500 m from the ditch, drawdown and flow are given by

$s = s_o E_1$, $q = q_o E_2$ with E_1 and E_2 function of the parameter u

$u = \frac{1}{2} \sqrt{\frac{\mu}{kH}} \frac{x}{\sqrt{t}}$ At t = 2 months = $(5.26)10^6$ sec

$$q_o = \frac{s_o}{\sqrt{\pi}} \sqrt{\mu kH} \frac{1}{\sqrt{t}} = \frac{3.5}{\sqrt{\pi}} \sqrt{(0.20)(9)10^{-3}} \frac{1}{\sqrt{(5.26)10^6}} \quad \text{or}$$

$$q_o = (36.5)10^{-6} \ m^3/m'/sec$$

$$u = \frac{1}{2} \sqrt{\frac{0.20}{(9)10^{-3}}} \frac{500}{\sqrt{(5.26)10^6}} = 0.514$$

$E_1(0.514) = 0.467$ $\qquad$ $E_2(0.514) = 0.768$

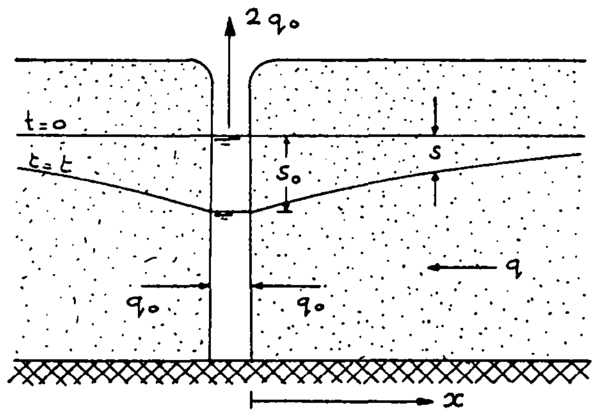
$s = (3.5)(0.467) = 1.63$ m, $\quad q = (36.5)10^{-6}\ (0.768)$ or

$q = (28.0)10^{-6}\ m^3/m'/sec$

2.22 An unconfined aquifer of infinite extend is situated above a horizontal impervious base. The coefficient of transmissibility kH of this aquifer amounts to $(18)10^{-3}$ m^2/sec, its specific yield μ to 25%.

Starting at t = o groundwater is abstracted from this aquifer by means of a fully penetrating ditch (of negligeable width) in an amount of $(30)10^{-6}$ m^3/m'/sec. At t = 10 days this abstraction is suddenly increased to $(50)10^{-6}$ m^3/m'/sec.

What is the lowering of the water level in the ditch at t = 30 days and what is at this moment the drawdown at a distance of 100 m from the ditch?



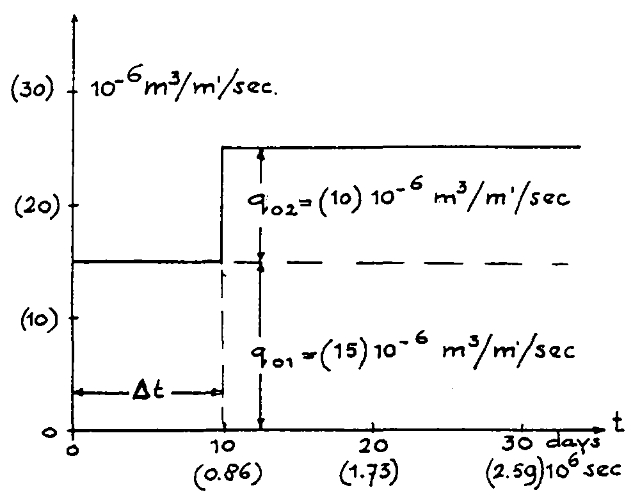
With the notations as indicated in the figure at the right, a sudden increase in the capacity of the gallery results in drawdowns given by

x = 0 $$s_o = \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t}$$

x = x $s = s_o E_3$ with E_3

function of the parameter

$$u = \frac{1}{2}\sqrt{\frac{\mu}{kH}} \frac{x}{\sqrt{t}}$$



The pattern of abstraction may be schematized as indicated in the diagram above, giving for t > Δt as lowering of the water level in the ditch

$$s_o = \frac{2q_{01}}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t} + \frac{2q_{02}}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t - \Delta t}$$

With the data under consideration and

$$t = 30 \text{ days} = (2.59)10^6 \text{ sec}, \Delta t = 10 \text{ days} = (0.86)10^6 \text{ sec}$$

this lowering becomes

$$s_o = \frac{(30)10^{-6}}{\sqrt{\pi}} \frac{1}{\sqrt{(0.25)(18)10^{-3}}} \sqrt{(2.59)10^6} + \frac{(20)10^{-6}}{\sqrt{\pi}} \times \frac{1}{\sqrt{(0.25)(18)10^{-3}}} \sqrt{(1.73)10^6}$$

$$s_o = 0.41 + 0.22 = 0.63 \text{ m}$$

At the same moment but at a distance x = 100 m from the ditch, the parameters u become

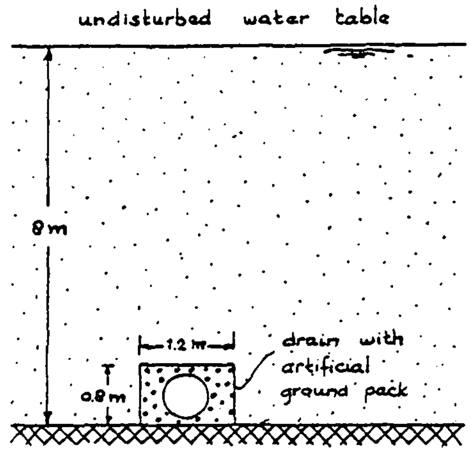
$$u_1 = \frac{1}{2} \sqrt{\frac{0.25}{(18)10^{-3}}} \frac{100}{\sqrt{(2.59)10^6}} = 0.116$$

$$u_2 = \frac{1}{2} \sqrt{\frac{0.25}{(18)10^{-3}}} \frac{100}{\sqrt{(1.73)10^6}} = 0.142, \quad \text{giving as drawdown}$$

$$s_{100} = (0.41)\ E_3\ (0.116) + (0.22)\ E_3\ (0.142)$$

$$s_{100} = (0.41)(0.808) + (0.22)(0.768) = 0.33 + 0.17 = 0.50 \text{ m}$$

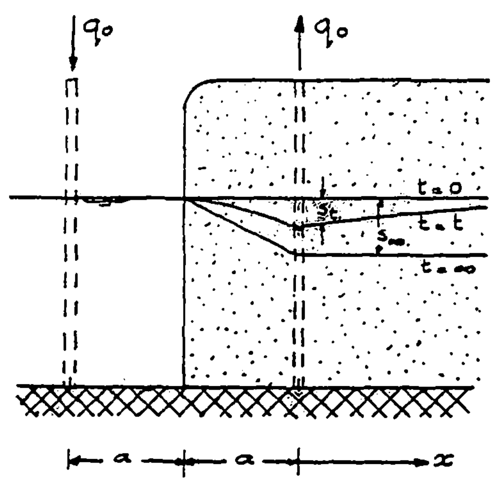
2.23 A semi-infinite unconfined aquifer is situated above an impervious base and bounded by a fully penetrating ditch with a constant and uniform waterlevel. The coefficient of permeability k of the aquifer amounts to $(0.25)10^{-3}$ m/sec, the saturated thickness H to 8 m and the specific yield μ to 15%.



At a distance of 50 m parallel to the ditch a gallery is constructed, the dimensions and position of which are shown in the picture at the right. Starting at t = 0 water is abstracted from this gallery in an amount of $(35)10^{-6}$ $m^3/m'/day$.

What is the lowering of the water level in the gallery after 10 days and how much time must elapse before 90% of the steady state drawdown is obtained?

At the boundary between the aquifer and the ditch with constant water level, the drawdown due to pumping the gallery remains zero. Mathematically this can be obtained by projecting an imaginary recharge gallery of the same capacity an equal distance at the other side of the shore line.



The drawdown due to pumping a fully penetrating gallery in an aquifer of infinite extent for a period of t days equals

$$s_o = \frac{q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu k H}} \sqrt{t}, \quad s_x = s_o E_3 \quad \text{with } E_3 \text{ a function of}$$

$$u = \frac{1}{2} \sqrt{\frac{\mu}{kH}} \frac{x}{\sqrt{t}}$$

In case under consideration, the drawdown at the face of the gallery at time t thus becomes

$$s_o = \frac{q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu k H}} \sqrt{t} \; \{1 - E_3(u_{2a})\}$$

With the data under consideration

$$u_{2a} = \frac{1}{2} \sqrt{\frac{0.15}{(0.25)10^{-3}(8)}} \frac{(2)(50)}{\sqrt{t}} = \frac{433}{\sqrt{t}}$$

$$s_o = \frac{(35)10^{-6}}{\sqrt{\pi}} \frac{1}{\sqrt{(0.15)(0.25)10^{-3}(8)}} \sqrt{t} \; \{1 - E_3 \left(\frac{433}{\sqrt{t}}\right)\}$$

$$s_o = (1.14)10^{-3} \sqrt{t} \; \{1 - E_3 \left(\frac{433}{\sqrt{t}}\right)\}$$

At $t = 10 \text{ days} = (0.864)10^6$ sec

$$s_o = (1.14)\sqrt{0.864} \; \{1 - E_3\left(\frac{0.433}{\sqrt{0.864}}\right)\} = 1.06\{1 - E_3(0.466)\}$$

$$s_o = 1.06 \; (1 - 0.384) = 0.65 \text{ m}$$

The calculations above in the meanwhile supposed a fully penetrating gallery. Due to partial penetration, an additional drawdown will result

$$\Delta s_o = \frac{q_o}{\pi k} \ln \frac{H}{\Omega} \quad \text{with } \Omega \text{ as wetted circumference}$$

$$\Omega = 1.2 + 2(0.8) = 2.8 \text{ m}$$

$$\Delta s_o = \frac{(35)10^{-6}}{\pi(0.25)10^{-3}} \ln \frac{8}{2.8} = 0.044 \ln 2.86 = 0.05 \text{ m}$$

giving as total drawdown after 10 days

$$s_o + \Delta s_o = 0.65 + 0.05 = 0.70 \text{ m}$$

The steady state drawdown equals

$$s_\infty = \frac{q_o}{kH} a + \Delta s_o = \frac{(35)10^{-6}}{(0.25)10^{-3}(8)} (50) + 0.05 =$$

$$= 0.88 + 0.05 = 0.93 \text{ m}$$

90% of this value or 0.84 m is reached at time t determined by

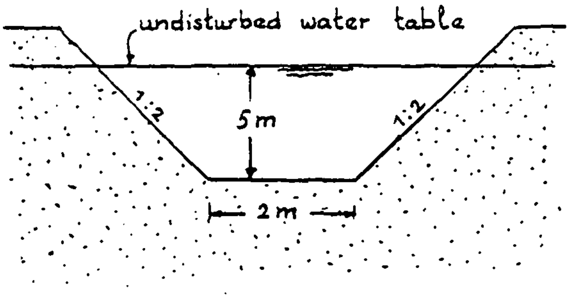
$$s_o = 0.84 - 0.05 = (1.14)10^{-3}\sqrt{t}\ \{1 - E_3\ (\frac{433}{\sqrt{t}})\}\ \text{or}$$

$$\frac{\sqrt{t}}{433}\ \{1 - E_3\ (\frac{433}{\sqrt{t}})\} = 1.60\ , \quad \text{from which follows}$$

$$\frac{\sqrt{t}}{433} = 5.8\ \text{or}\ \ t = (6.3)10^6\ \text{sec} = 73\ \text{days}$$

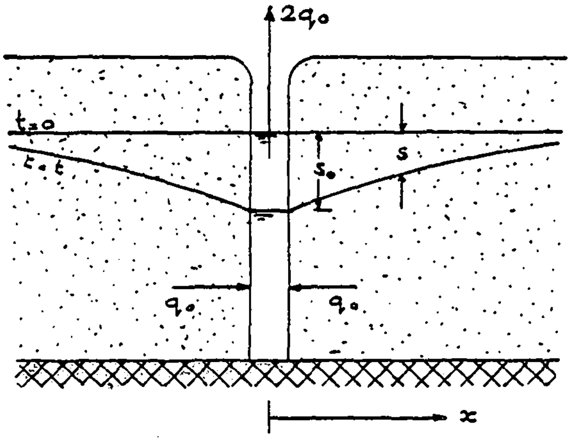
2.24 An unconfined aquifer of infinite extent is situated above a horizontal impervious base. The coefficient of permeability k of this aquifer amounts to $(0.48)10^{-3}$ m/sec, the saturated thickness H to 25 m and the specific yield μ to 30%.

In this aquifer a ditch is constructed, the shape and dimensions of which are shown in the picture at the right. During a period of 1 month water is abstracted from this ditch in an amount of $(0.20)10^{-3}$ $m^3/m'/sec$.



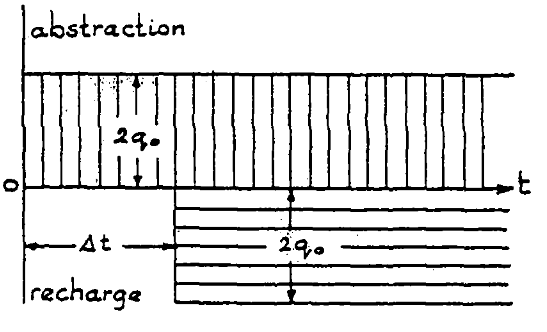
What is the water level in the ditch at the end of the pumping period and 6 months after abstraction has stopped? What is the maximum lowering of the groundwater table in a point 500 m from the ditch?

When provisionnally the partially penetrating ditch is replaced by a fully penetrating gallery - as shown in the picture at the right, an abstraction $2q_o$ starting at t = 0 will lower the waterlevel in the gallery by



$$s_o = \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t}$$

In mathematical respect, cessation of pumping can only be obtained by superimposing a recharge of the same magnitude



$$t > \Delta t \quad s_o = \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t} - \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t - \Delta t}$$

With the data under consideration

$$s_o = \frac{(0.20)10^{-3}}{\sqrt{\pi}} \frac{1}{\sqrt{(0.30)(0.48)10^{-3}\ (25)}} (\sqrt{t} - \sqrt{t - \Delta t})$$

$$s_o = (1.88)10^{-3} (\sqrt{t} - \sqrt{t - \Delta t})$$

At the end of the pumping period

$$t = 1 \text{ month} = (2.63)10^6 \text{ sec}, \quad t - \Delta t = 0 \quad \text{and}$$

$$s_o = (1.88)10^{-3} \sqrt{(2.63)10^6} = 3.05 \text{ m}$$

Six months after abstraction has stopped

$$t = 7 \text{ months} = (18.4)10^6 \text{ sec},$$
$$t - \Delta t = 6 \text{ months} = (15.8)10^6 \text{ sec}$$

$$s_o = (1.88)10^{-3} \{\sqrt{(18.4)10^6} - \sqrt{(15.8)10^6}\} =$$
$$= (1.88)(4.29 - 3.97) = 0.60 \text{ m}$$

In reality, however, the ditch only partially penetrates the aquifer, from which circumstance an additional drawdown will result.

$$\Delta s_o = \frac{2q_o}{\pi k} \ln \frac{H - s_o}{\Omega} \quad \text{with } \Omega \text{ as wetted circumference.}$$

With the total lowering $s_o + \Delta s_o$ estimated at 3.2 m, the remaining water table depth in the ditch equals 1.8 m, giving

$$\Omega = 2\sqrt{5}\,(1.8) + 2 = 10.1 \text{ m}$$

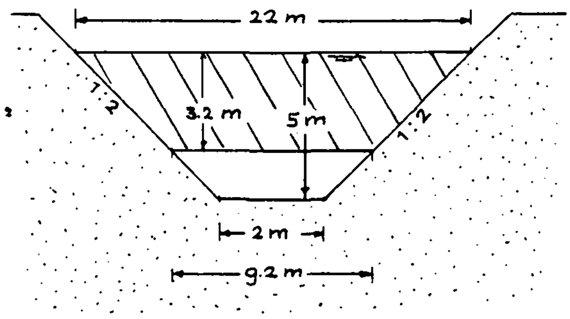
$$\Delta s_o = \frac{(0.20)10^{-3}}{\pi(0.48)10^{-3}} \ln \frac{40 - 3.1}{10.1} = 0.133 \ln 3.65 = 0.17 \text{ m} \quad \text{or}$$

$$s_o + \Delta s_o = 3.05 + 0.17 = 3.2 \text{ m}$$

When abstraction stops, the additional drawdown disappears and for the remaining drawdown after 6 months, no correction is necessary.

The abstraction of $(0.2)10^{-3}$ $m^3/m'/sec$ during 1 month = $(2.63)10^6$ sec ads up to 526 m^3/m'.

Part of this water in the meanwhile comes from the ditch itself. With the shaded area in the picture at the right equal to 50 m^2 = 50 m^3/m', only 526 - 50 = 476 m^3/m' comes from the aquifer proper. This reduces the drawdown after 1 month to about



$$s_o' = \frac{476}{526} 3.2 = 2.9 \text{ m}$$

In the same way the drawdown 6 months after abstraction has stopped must be reduced from 0.6 m to

$$s_o' = \frac{514}{526} 0.60 = 0.59 \text{ m}, \quad \text{the difference being negligeable.}$$

In a point at a distance x from the ditch, at a time t > Δt, the drawdown equals

$$s = \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \{\sqrt{t}\, E_3(u_t) - \sqrt{t - \Delta t}\, E_3(u_{t - \Delta t})\} \quad \text{with}$$

$$u_t = \frac{1}{2} \sqrt{\frac{\mu}{kH}} \frac{x}{\sqrt{t}} \qquad u_{t - \Delta t} = \frac{1}{2} \sqrt{\frac{\mu}{kH}} \frac{x}{\sqrt{t - \Delta t}}$$

$$E_3(u) = e^{-u^2} - \sqrt{\pi}\, u + 2u \int_0^u e^{-u^2}\, du$$

This drawdown reaches its maximum value for

$$\frac{ds}{dt} = \frac{ds}{du} \frac{du}{dt} = 0$$

from which follows as time - distance relationship for maximum drawdown

$$\frac{(t)(t - \Delta t)}{\Delta t} \ln \frac{t}{t - \Delta t} = \frac{1}{2} \frac{\mu}{kH} x^2$$

With Δt = 1 month = $(2.63)10^6$ sec, x = 500 m and

$$\frac{1}{2} \frac{\mu}{kH} x^2 = \frac{1}{2} \frac{0.3}{(0.48)10^{-3}\ (25)} (500)^2 = (3.13)10^6$$

the time of maximum drawdown t can be calculated at $(4.79)10^6$ sec or 1.82 months. At this moment

$$u_t = \frac{1}{2} \sqrt{\frac{0.3}{(0.48)10^{-3}\ (25)}} \frac{500}{\sqrt{(4.79)10^6}} = 0.572\ ,\ E_3 = 0.297$$

$$u_{t\ -\ \Delta t} = \frac{1}{2} \sqrt{\frac{0.3}{(0.48)10^{-3}\ (25)}} \frac{500}{\sqrt{(2.16)10^6}} = 0.848\ ,\ E_3 = 0.141$$

With an effective abstraction of 476 m^3/m'/month = $(0.181)10^{-3}$ m^3/m'/sec the maximum drawdown at a distance of 500 m from the gallery becomes

$$s = \frac{(0.181)10^{-3}}{\sqrt{\pi}} \frac{1}{\sqrt{(0.3)(0.48)10^{-3}\ (25)}} \times$$

$$\{\sqrt{(4.79)10^6}\ (0.297) - \sqrt{(2.16)10^6}\ (0.141)\}$$

$$s = (1.70)10^{-3}\ \{(0.65)10^{\ 3} - (0.21)10^3\} = 0.75\ m$$

2.25 A semi-infinite unconfined aquifer is situated above an impervious base and bounded by a lake. The coefficient of permeability k of the aquifer amounts to $(0.6)10^{-3}$ m/sec, its specific yield μ to 23%. To store water during autumn and winter, the lake level rises linearly from 16 to 20 m above the impervious base. This water is used during spring and summer, causing the lake level to drop linearly by the same amount.

What is the additional amount of groundwater storage obtained by the flow of lake water into and out from the aquifer?

When for $t < 0$ the lake level is constant, while for $t \geqslant 0$ this level shows a linear rise

$$s = \alpha t$$

then the rate at which lake water enters the aquifer equals

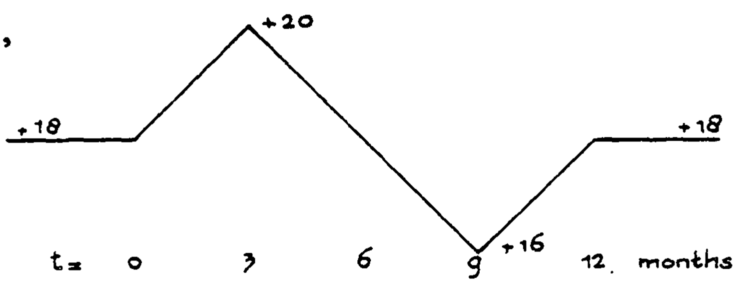
$$q = \frac{2\alpha}{\sqrt{\pi}} \sqrt{\mu k H} \sqrt{t}$$

With t expressed in months (1 month = $(2.63)10^6$ sec) and the other factors as mentioned above

$$q = \frac{2}{\sqrt{\pi}} \frac{20 - 16}{(6)(2.63)10^6} \sqrt{(0.23)(0.6)10^{-3} \frac{20+16}{2}} \sqrt{(2.63)10^6\, t}$$

$$q = (0.231)10^{-4}\sqrt{t}\ \text{m}^3/\text{m}'/\text{sec}$$

For the lakelevel variation as sketched at the right, the inflows thus become



$0 < t < 3$ months	$q = 0.231\sqrt{t}$
$3 < t < 9$	$q = 0.231\sqrt{t} - 0.462\sqrt{t-3}$
$9 < t < 12$	$q = 0.231\sqrt{t} - 0.462\sqrt{t-3} + 0.462\sqrt{t-9}$
$12 < t$	$q = 0.231\sqrt{t} - 0.462\sqrt{t-3} + 0.462\sqrt{t-9} - 0.231\sqrt{t-12}$

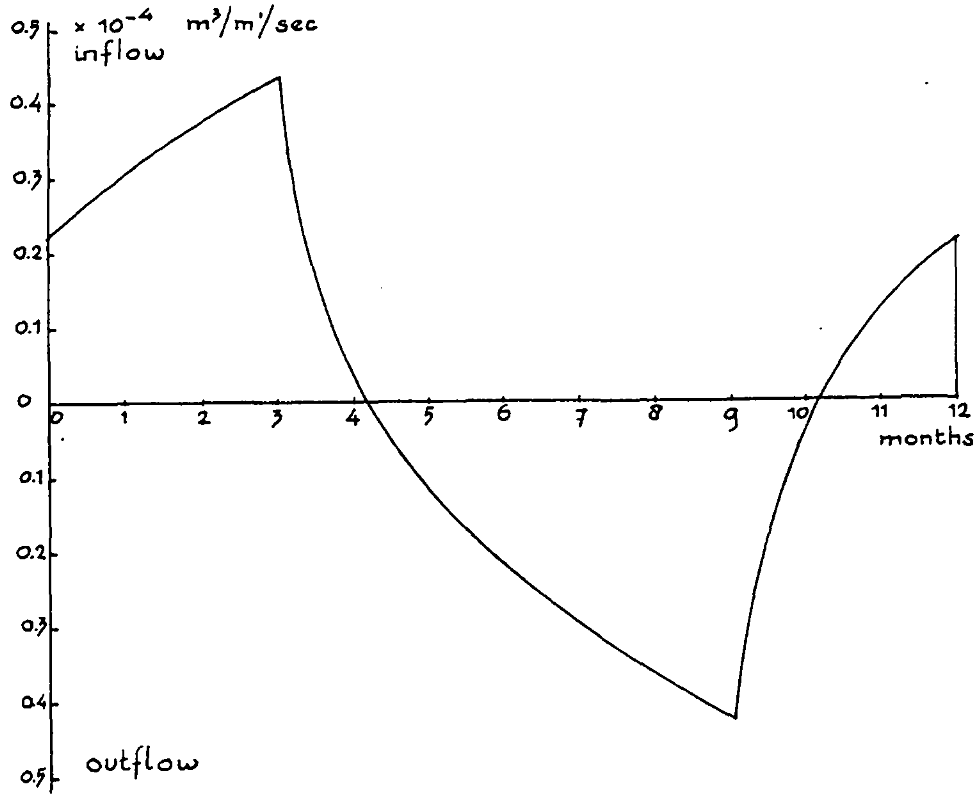
This gives

t	10^4 q	t	10^4 q	t	10^4 q	t	10^4 q	t	10^4 q
0	0.000	12	+ 0.214	24	+ 0.004	36	+ 0.001	0	+ 0.219
1	+ 0.231	13	+ 0.065	25	+ 0.003	37	+ 0.001	1	+ 0.300
2	+ 0.327	14	+ 0.039	26	+ 0.003	38	+ 0.001	2	+ 0.370
3	+ 0.401	15	+ 0.026	27	+ 0.002	39	+ 0.001	3	+ 0.430
4	0.000	16	+ 0.019	28	+ 0.002	40	+ 0.001	4	+ 0.022
5	- 0.137	17	+ 0.014	29	+ 0.002	41	+ 0.001	5	- 0.120
6	- 0.234	18	+ 0.011	30	+ 0.002	42	+ 0.001	6	- 0.220
7	- 0.313	19	+ 0.009	31	+ 0.002	43	+ 0.001	7	- 0.301
8	- 0.380	20	+ 0.007	32	+ 0.001	44	+ 0.001	8	- 0.371
9	- 0.439	21	+ 0.006	33	+ 0.001	45	+ 0.000	9	- 0.432
10	- 0.030	22	+ 0.005	34	+ 0.001	46	+ 0.000	10	- 0.024
11	+ 0.113	23	+ 0.004	35	+ 0.001	47	+ 0.000	11	+ 0.118
12	+ 0.214	24	+ 0.004	36	+ 0.001	48	+ 0.000	12	+ 0.219

In reality, however, the cycle is not restricted to one year, but it repeats itself indefinitely. Mathematically this can be taken into account by applying the method of superposition.

$$q_t = q_t + q_{t+12} + q_{t+24} + q_{t+36} + \ldots..$$

The results are shown in the table above, at the far right. Due to unavoidable errors in calculation, the absolute value of the flows q_n and q_{n+6} are not exactly the same - as they ought to be - but the differences are negligeable small. Graphically the exchange between lake and aquifer is shown at page 2.25-c, from which follows



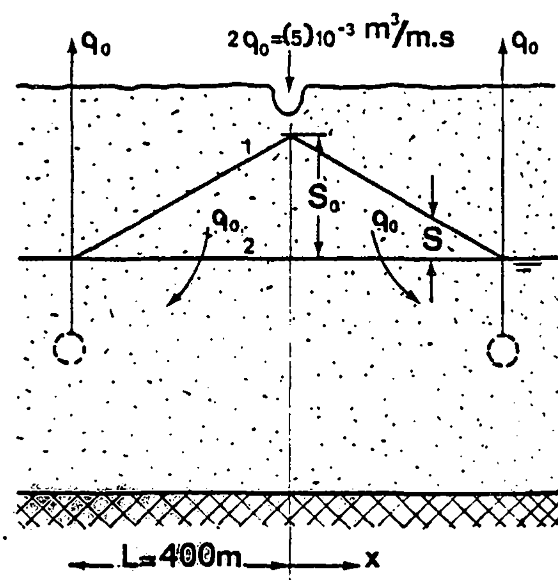
as total in- or outflow during a 6 months period

$$\Sigma q = 380 \ m^3/m'$$

This amount is equivalent to the lake storage over an additional width of 95 m !

2.26 An unconfined aquifer of infinite extent is situated above an impervious base. Its coefficient of transmissibility equals 0.08 m^2/sec, while its specific yield μ amounts to 40%. In this aquifer 3 parallel galleries are constructed, at equal intervals of 600 m. The centre gallery is used for artificial recharge of the aquifer in an amount of $(5)10^{-3}$ $m^3/m,sec$, while the same amount of water is recovered by the outer galleries.

What is the rise of the water table under the center ditch during steady-state operation and how far will this water level drop when recharge is interrupted for 6 weeks while abstraction continues at the same rate?



For steady-state conditions, the equations of flow are

Darcy $$q = -kH\frac{ds}{dx}$$

continuity $$q = q_o$$

combined $$ds = -\frac{q_o}{kH}dx$$

integrated $$s = -\frac{q_o}{kH}x + C_1$$

boundary condition $x = L$, $s = o$

$$o = -\frac{q_o}{kH}L + C_1$$

combined $$s = \frac{q_o}{kH}(L - x),$$

and $$s_o = \frac{q_o}{kH}L$$

With the data under consideration

$$s_o = \frac{(2.5)10^{-3}}{0.08}600 = 18.75 \text{ m}$$

Mathematically spoken, interruption of recharge can easiest be accomplished by superimposing an abstraction of magnitude $2q_o$ from the recharge ditch. This lowers the water level at the recharge ditch by

$$s'_o = \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu K H}} \sqrt{t}$$

This gives after 6 weeks = $(3.63)10^6$ sec

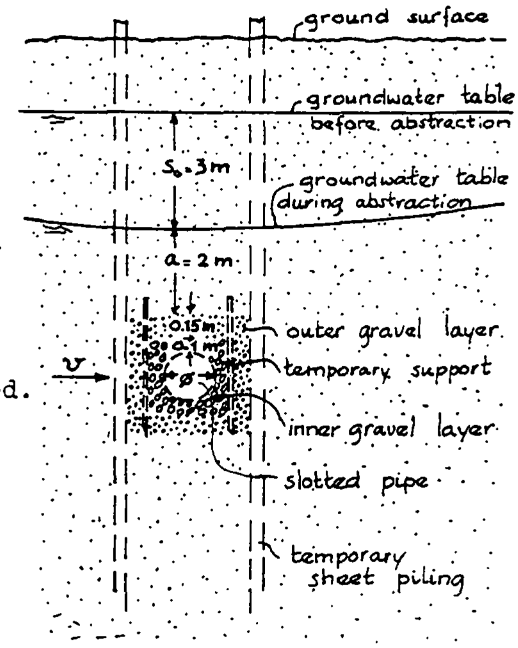
$$s'_o = \frac{(5)10^{-3}}{\sqrt{\pi}} \frac{1}{\sqrt{(0.4)(0.08)}} \sqrt{(3.63)10^6} = 30.05 \text{ m}$$

that is to 30.05 - 18.75 = 11.3 m below the original groundwater table.

3.11 From an unconfined aquifer, composed of sand with a coefficient of permeability k equal to $(0.25)10^{-3}$ m/sec, a drain with a length of 1600 m abstracts groundwater in an amount Q_o of 0.2 m^3/sec. Due to this abstraction a lowering s_o of the groundwater table by 3 m must be expected.

Sketch the drain construction to be applied and indicate the most important dimensions.

The coefficient of permeability k equal to $(0.25)10^{-3}$ m/sec is fairly low, pointing to a rather fine sand. With regard to the danger of clogging, porous drains are not advisable. Slotted drains must be prefered, while to obtain slots as wide as possible, a double gravel treatment will be applied. The general construction is shown in the sketch at the right, with the top of the gravel pack a distance of at least 2 m below the lowest waterlevel during operation.



The outside dimensions of the gravel pack primarily depends on the maximum allowable entrance velocity v_a. For vertical wells Sichardt gives

$$v_a = \frac{\sqrt{k}}{30}$$

Once clogged, drains cannot be cleaned, asking for an additional factor of safety, lowering the maximum allowable entrance velocity to

$$v_a = \frac{\sqrt{k}}{60}$$ In the case under consideration

$$v_a = \frac{\sqrt{(0.25)10^{-3}}}{60} = (0.26)10^{-3} \text{ m/sec}$$

The abstraction per unit length of drain equals in average

$$q_o = \frac{Q_o}{L} = \frac{0.2}{1600} = (0.125)10^{-3} \text{ m}^3\text{/m'/sec}$$ asking for a

minimum circumference Ω of the gravel pack equal to

$$\Omega = \frac{q_o}{v_a} = \frac{(0.125)10^{-3}}{(0.26)10^{-3}} = 0.5 \text{ m}'$$

This means a square with sides of $\frac{0.5}{4}$ = 0.125 m. With an artificial gravel pack such a size is always present and the maximum allowable entrance velocity is not a deciding factor.

With regard to the velocity of lateral flow, the inside diameter of the slotted drain must satisfy two contradictory requirements. On one hand this velocity must be large so as to be self-cleaning, while on the other hand the velocity must be small to keep the friction losses down, which otherwise would result in a rather uneven abstraction of groundwater over the length of the gallery. After careful consideration it is decided to keep the velocity between the limits of 0.3 and 0.6 m/sec, increasing the diameter of the drain stepwise as indicated below

Q	φ	v	L	l
0.2 m^3/sec	0.7 m	0.52 m/sec	1600 m	670 m
0.116	0.7	0.30	930	
0.116	0.5	0.59	930	460
0.059	0.5	0.30	470	
0.059	0.35	0.61	470	240
0.029	0.35	0.30	230	
0.029	0.25	0.59	230	230
0.000	0.25	0.00	0	

that is to say 4 sections with

length	230	240	460	670 m
diameter	0.25	0.35	0.5	0.7 m ø
size gravel pack	0.75	0.85	1.0	1.2 m ◻

With regard to the grain size distribution of the gravel pack, it is first considered that according to Allan Hazen the coefficient of permeability and the effective diameter of the aquifer material are interrelated by

$$k = (11)10^3 \; d_{10}^2 \qquad \text{or}$$

$$d_{10} = \sqrt{\frac{k}{(11)10^3}} = \sqrt{\frac{(0.25)10^{-3}}{(11)10^3}} = (0.15)\; 10^{-3} \text{ m}$$

The coefficient of uniformity remains unknown, but it may safely be assumed, that the 85% diameter exceeds 0.3 mm. With the lower limit of the outer gravel layer a factor 4 larger than this 85% diameter and the upper limit a factor $\sqrt{2}$ coarser than the lower limit, this gives

outer gravel layer 1.2 - 1.7 mm

The inner gravel layer is again a factor 4 coarser or

inner gravel layer 5 - 7 mm, allowing as

slot width 2 mm

When the entrance velocity inside the slots is limited to $(10)10^{-3}$ m/sec, the length of slot per m' of drain equals

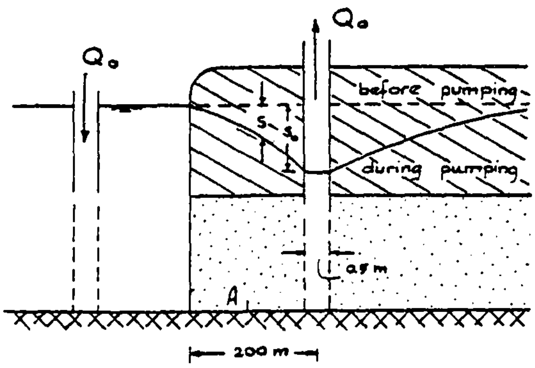
$$(2)10^{-3}\, b = \frac{q_o}{(10)10^{-3}} = \frac{(0.125)10^{-3}}{(10)10^3} \qquad \text{or} \quad b = 6.25 \text{ m}$$

With the smallest drain diameter of 0.25 m and perpendicular slots over the lower half only, this means a number of slots per m' drain equal to

$$n = \frac{6.25}{(0.5)\pi(0.25)} = 16$$

4.01 A semi-infinite confined aquifer without recharge from above or from below is bounded by a fully penetrating ditch and has a coefficient of transmissibility kH equal to $(3)10^{-3}$ m^2/sec. At a distance of 200 m from the stream a fully penetrating well with an outside diameter of 0.5 m is pumped at a constant rate of $(7)10^{-3}$ m^3/sec.

What is the drawdown at the well face and what is the drawdown in a point A halfway between the well and the shoreline?



With the method of images, the drawdown in a point at a distance r from the well and a distance r' from the imaginary recharge well is given by

$$s = \frac{Q_o}{2\pi kH}\{\ln\frac{R}{r} - \ln\frac{R}{r'}\} = \frac{Q_o}{2\pi kH}\ln\frac{r'}{r}$$

With r' = 2L, the drawdown at the well face becomes

$$s_o = \frac{(7)10^{-3}}{2\pi(3)10^{-3}}\ln\frac{400}{0.25} = 0.371 \ln 1600 = 2.74 \text{ m}$$

and in point A, at a distance r from the well

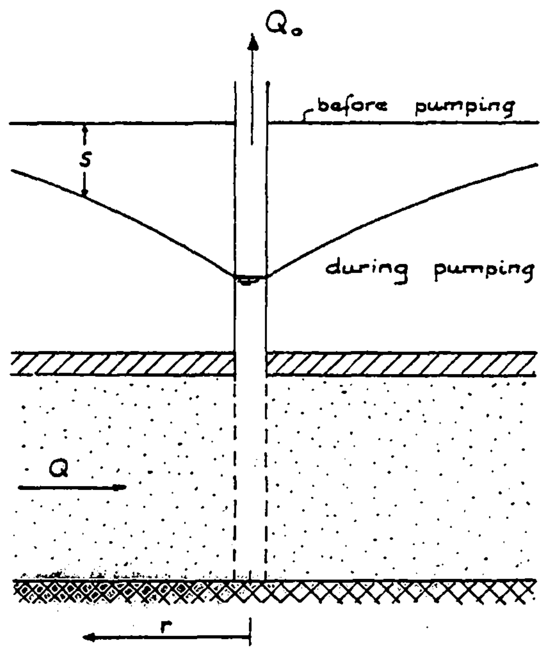
$$s_A = \frac{Q_o}{2\pi kH}\ln\frac{2L - r}{r}$$

$$s_A = 0{,}371 \ln\frac{300}{100} = 0{,}41 \text{ m}$$

4.02 A leaky artesian aquifer is situated between an impervious base and an overlying less-pervious layer. Above the latter layer an unconfined aquifer with a constant and uniform water level is present. The coefficient of transmissibility $k\bar{H}$ of the artesian aquifer amounts to $(2.5)10^{-3}\,m^2/sec$, the resistance c of the less-pervious layer against vertical water movement to $(40)10^6$ sec.

From the artesian aquifer a fully penetrating well with an outside diameter of 0.4 m abstracts water at a rate of $(6)10^{-3}\ m^3/sec$.

What is the lowering of the artesian water table at a distance of 1000, 100, 10 and 1 m from the well centre and at the well face? How much water infiltrates from above within a radius of 200 m around the well?



For a well in a leaky artesian aquifer of infinite extent, the drawdown equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \quad \text{with}$$

$$\lambda = \sqrt{kHc}$$

In case $\frac{r}{\lambda}$ is small, the drawdown may be approximated by

$$s = \frac{Q_o}{2\pi kH} \ln \frac{1.123\lambda}{r}$$

With the data as given

$$\lambda = \sqrt{(2.5)10^{-3}(40)10^6} = 316 \text{ m}$$

$$s = \frac{(6)10^{-3}}{2\pi(2.5)10^{-3}} K_o\left(\frac{r}{316}\right) \approx \frac{(6)10^{-3}}{2\pi(2.5)10^{-3}} \ln \frac{(1.123)(316)}{r}$$

$$s = 0.382\, K_o\left(\frac{r}{316}\right) \approx 0.382 \ln \frac{355}{r}$$

This gives as drawdowns

$r =$	1000	100	10	1	0.2 m
$K_o(\frac{r}{316}) =$	0.029	1.32	3.58	-	-
$\ln \frac{355}{r} =$	-	1.27	3.57	5.87	7.48
$s =$	0.01	0.50	1.37	2.24	2.86 m

At a distance r from the well centre, the rate of flow equals

$$Q = Q_o \frac{r}{\lambda} K_1 \left(\frac{r}{\lambda}\right) \qquad \text{At } r = 200 \text{ m}$$

$$Q = Q_o \frac{200}{316} K_1 \left(\frac{200}{316}\right) = Q_o(0.633)\ K_1\ (0.633) = Q_o(0.633)(1.21)=0.766\ Q_o$$

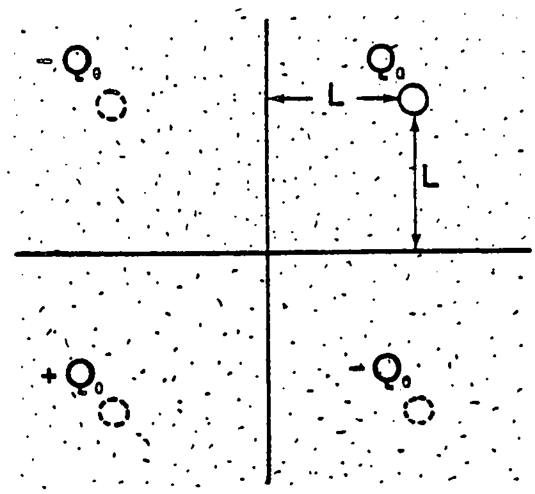
The difference between this rate and the amount Q_o abstracted by the well, infiltrates from above

$$I = Q_o - 0.766\ Q_o = (0.234)Q_o = (0.234)(6)10^{-3} = (1.4)10^{-3}\ m^3/sec.$$

4.03 A leaky artesian aquifer is situated above an impervious base, has a coefficient of transmissibility kH equal to 0.012 m^2/sec and is overlain by a semi-pervious layer with a resistance of $(30)10^6$ sec against vertical water movement.

The aquifer is crossed by two fully penetrating ditches, intersecting each other perpendicularly. In the aquifer a fully penetrating well is set, with an outside diameter of 0.4 m at equal distances of 500 m from both ditches. The well is pumped at a capacity of 0.035 m^3/sec.

What is the influence of those ditches on the drawdown at the well face?



For a well in a leaky artesian aquifer of infinite extent the drawdown equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \quad \text{with}$$

$$\lambda = \sqrt{kHc} = \sqrt{(0.012)(30)10^6} = 600 \text{ m}$$

At the well face this may be approximated by

$$s_o = \frac{Q_o}{2\pi kH} \ln \frac{1.123\ \lambda}{r_o}$$, in the case under consideration

$$s_o = \frac{0.035}{2\pi(0.012)} \ln \frac{(1.123)(600)}{0.2} = 0.464 \ln 3369 = 3.77 \text{ m}$$

Using the method of images, the drawdown at the well face in a semi-infinite quadrant becomes

$$\bar{s}_o = \frac{Q_o}{2\pi kH} \left\{ \ln \frac{1.123\ \lambda}{r_o} - 2K_o\left(\frac{2L}{\lambda}\right) + K_o\left(\frac{2L\sqrt{2}}{\lambda}\right) \right\}$$

$$\bar{s}_o = 0.464 \left\{ \ln \frac{(1.123)(600)}{0.2} - 2K_o\left(\frac{1000}{600}\right) + K_o\left(\frac{1000\sqrt{2}}{600}\right) \right\}$$

$$s_o = 0.464 \left\{ \ln 3369 - 2K_o(1.667) + K_o(2.357) \right\}$$

$$s_o = 0.464 \{ 8.122 - (2)(0.1726) + 0.0739 \}$$

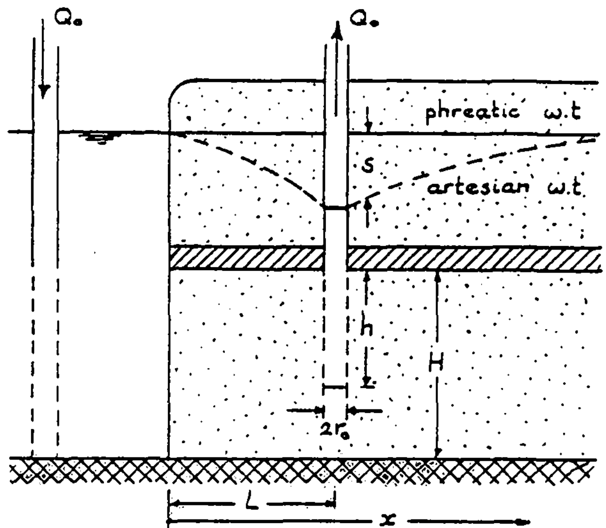
$$s_o = (0.464)(7.851) = 3.64 \text{ m}$$

that is to say a difference of 0.13 m or 3,5%.

4.04 A semi-infinite leaky artesian aquifer has a thickness of 50 m and a coefficient of permeability k equal to $(0.40)10^{-3}$ m/sec. The aquifer is situated above an impervious base and overlain by a semi-pervious layer with a resistance C of $(200)10^{6}$ sec against vertical water movement. The water level in the unconfined aquifer above the semi-pervious layer is uniform and constant, equal to the water level in the bounding ditch.

At a distance of 500 m from the ditch a partially penetrating well is constructed. Its outside diameter equals 0.6 m, while the screen extends from the top of the aquifer 30 m downward. The well is pumped at a constant rate of $(30)10^{-3}$ m^3/sec.

What is the drawdown of the artesian water table at the well face and at what distance from the ditch does this drawdown decline to 0.1 m?



Using the method of images gives as drawdown at the face of the fully penetrating well

$$s_o = \frac{Q_o}{2\pi kH} \{K_o(\frac{r_o}{\lambda}) - K_o(\frac{2L}{\lambda})\} \quad \text{in which}$$

$$\lambda = \sqrt{kHc} = \sqrt{(0.40)10^{-3}\ (50)(200)10^{6}} = 2000 \text{ m}$$

With $\frac{r_o}{\lambda} = \frac{0.3}{2000}$ small

$$K_o(\frac{r_o}{\lambda}) = \ln \frac{1.123\ \lambda}{r_o}$$ Substitution gives with the data supplied

$$s_o = \frac{(30)10^{-3}}{2\pi(0.40)10^{-3}\ (50)} \{\ln \frac{(1.123)(2000)}{0.3} - K_o(\frac{1000}{2000})\}$$

$$s_o = (0.239)\{\ln 7490 - K_o(0.5)\} = 0.239\ (8.91 - 0.92) \quad \text{or}$$

$$s_o = 1.91 \text{ m}$$

In reality the well only partially penetrates the aquifer, resulting in an additional drawdown equal to

$$\Delta s_o = \frac{Q_o}{2\pi kH}\frac{1-p}{p}\ln\frac{(1-p)h}{r_o} \quad \text{with } p = \frac{h}{H} = \frac{30}{50} = 0.6$$

$$\Delta s_o = (0.239)\frac{0.4}{0.6}\ln\frac{(0.4)(30)}{0.3} = (0.239)(0.667)(3.69) = 0.59 \text{ m}$$

The total drawdown at the well face thus becomes

$$s_o + \Delta s_o = 1.91 + 0.59 = 2.5 \text{ m}$$

The largest drawdowns occur in a line through the well, perpendicular to the bounding ditch. At a distance x from the shoreline this drawdown equals

$$s = \frac{Q_o}{2\pi kH}\{K_o(\frac{x-L}{\lambda}) - K_o(\frac{x+L}{\lambda})\}$$

A drawdown of 0.1 m occurs at

$$0.1 = 0.239\{K_o(\frac{x-500}{2000}) - K_o(\frac{x+500}{2000})\} \quad \text{or}$$

$$0.418 = K_o(\frac{x-500}{2000}) - K_o(\frac{x+500}{2000})$$

With trial and error

x = 1000 m	$K_o(0.25) - K_o(0.75) = 1.542 - 0.611 = 0.931$
1500 m	$K_o(0.50) - K_o(1.00) = 0.924 - 0.421 = 0.503$
1800 m	$K_o(0.65) - K_o(1.15) = 0.716 - 0.341 = 0.375$

By interpolation

$$x = 1500 + \frac{0.503 - 0.418}{0.503 - 0.375}(300) = 1500 + 200 = 1700 \text{ m}$$

4.05 From below to above a geo-hydrological profile shows

an impervious base;

a waterbearing formation with a coefficient of transmissibility k_2H_2 equal to $(30)10^{-3}$ m^2/sec;

a semi-pervious layer with a resistance c_2 of $(50)10^6$ sec against vertical water movement;

a waterbearing formation with a coefficient of transmissibility k_1H_1 equal to $(6)10^{-3}$ m^2/sec;

a semi-pervious layer with a resistance c_1 of $(400)10^6$ sec against vertical water movement;

an unconfined aquifer with a constant and uniform water level.

From the upper artesian aquifer, groundwater is abstracted in an amount $Q_o = (28)10^{-3}$ m^3/sec by means of a fully penetrating well with a diameter of 0.4 m

Calculate the drawdown in the upper and lower artesian aquifers as function of the distance to the pumped well.

In the case of a two-layered leaky artesian aquifer and groundwater-abstraction from the upper storey, the drawdown formulae are

$$s_1 = \frac{Q_o}{2\pi k_1H_1}\frac{1}{\lambda_1 - \lambda_2}\{(\lambda_1 - \alpha_2)\ K_o\ (\sqrt{\lambda_1}r) + (\alpha_2 - \lambda_2)\ K_o\ (\sqrt{\lambda_2}r)\}$$

$$s_2 = \frac{Q_o}{2\pi k_1H_1}\frac{\alpha_2}{\lambda_1 - \lambda_2}\{-K_o\ (\sqrt{\lambda_1}r) + K_o\ (\sqrt{\lambda_2}r)\} \quad \text{with}$$

$$\alpha_1 = \frac{1}{k_1H_1c_1} \qquad \alpha_2 = \frac{1}{k_2H_2c_2} \qquad \beta_1 = \frac{1}{k_1H_1c_2} \qquad \text{and}$$

$$\begin{matrix}\lambda_1\\ \lambda_2\end{matrix} = \frac{1}{2}\{\alpha_1 + \alpha_2 + \beta_1 \pm \sqrt{(\alpha_1 + \alpha_2 + \beta_1)^2 - 4\alpha_1\alpha_2}\}$$

With the data under consideration

$$\alpha_1 = \frac{1}{(6)10^{-3}(400)10^6} = (0.417)10^{-6}$$

$$\alpha_2 = \frac{1}{(30)10^{-3}(50)10^6} = (0.667)10^{-6}$$

$$\beta_1 = \frac{1}{(6)10^{-3}(50)10^{6}} = (3.333)10^{-6}$$

$$\alpha_1 + \alpha_2 + \beta_1 = (4.417)10^{-6}$$

$$4\alpha_1\alpha_2 = (1.111)10^{-12}$$

$$\begin{matrix}\lambda_1\\ \lambda_2\end{matrix} = \frac{1}{2}\{(4.417)10^{-6} \pm \sqrt{(19.510)10^{-12} - (1.111)10^{-12}}\}$$

$$\lambda_1 = \frac{1}{2}\{(4.417)10^{-6} + (4.289)10^{-6}\} = (4.353)10^{-6}$$

$$\lambda_2 = \frac{1}{2}\{(4.417)10^{-6} - (4.289)10^{-6}\} = (0.064)10^{-6}$$

$$\sqrt{\lambda_1} = \frac{1}{480} \qquad \sqrt{\lambda_2} = \frac{1}{3950}$$

The drawdown formulae thus become

$$s_1 = \frac{(28)10^{-3}}{2\pi(6)10^{-3}} \frac{1}{(4.289)10^{-6}} \{(3.686)10^{-6} K_o(\frac{r}{480}) + (0.603)10^{-6} K_o(\frac{r}{3950})\}$$

$$s_2 = \frac{(28)10^{-3}}{2\pi(6)10^{-3}} \frac{(0.667)10^{-6}}{(4.289)10^{-6}} \{-K_o(\frac{r}{480}) + K_o(\frac{r}{3950})\}$$

Simplified

$$s_1 = 0.639\ K_o(\frac{r}{480}) + 0.105\ K_o(\frac{r}{3950})$$

$$s_2 = 0.116\ \{-K_o(\frac{r}{480}) + K_o(\frac{r}{3950})\}$$

In the vicinity of the well, the approximation

$K_o(\frac{r}{\lambda}) = \ln \frac{(1.123)\lambda}{r}$ may be applied, giving as drawdowns

$$s_1 = \frac{Q_o}{2\pi k_1 H_1} \{\ln \frac{1.123}{r} + \frac{(\lambda_1 - \alpha_2) \ln \frac{1}{\sqrt{\lambda_1}} + (\alpha_2 - \lambda_2) \ln \frac{1}{\sqrt{\lambda_2}}}{\lambda_1 - \lambda_2}\}$$

$$s_2 = \frac{Q_o}{2\pi k_1 H_1} \frac{\alpha_2}{\lambda_1 - \lambda_2} \ln \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}}$$

or in the case under consideration

$$s_1 = 0.742\ (\ln \frac{1.123}{r} + 6.47) = 0.742 \ln \frac{740}{r}$$

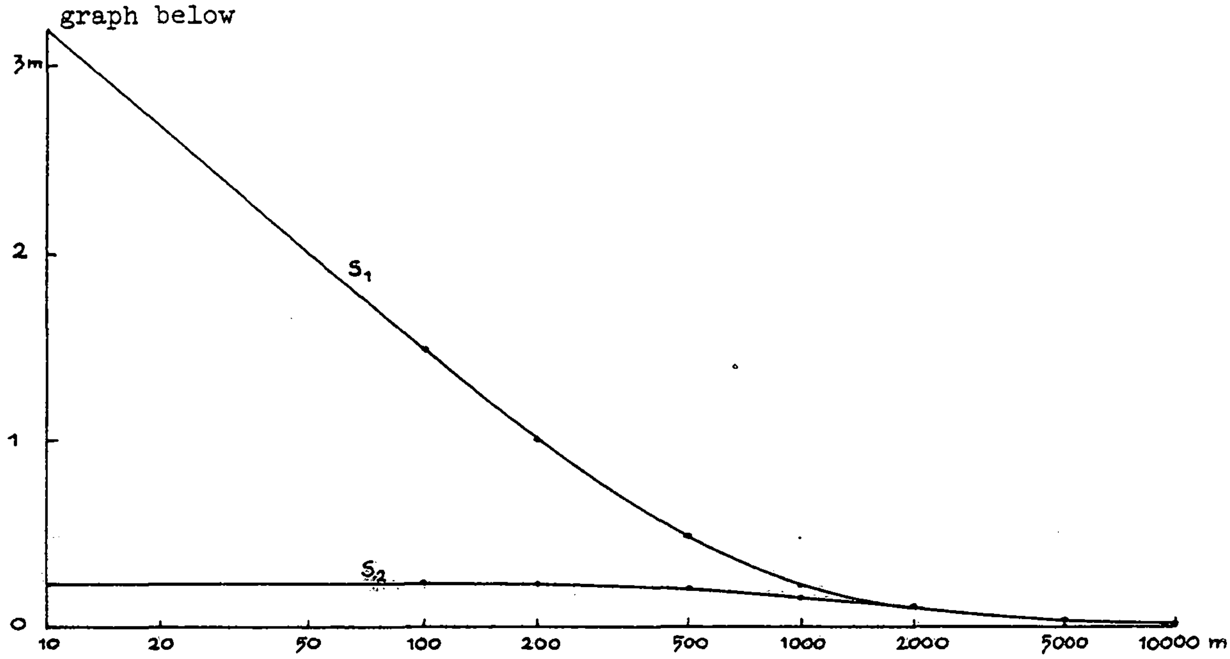
$$s_2 = 0.243$$

The drawdowns at the well face thus become

$$s_{01} = 0.742 \ln \frac{740}{0.2} = 6.10 \text{ m}$$

$$s_{02} = 0.24 \text{ m}$$

For greater distances from the well, the drawdowns are shown in the graph below



In the near vicinity of the well, the drawdown s_1 may be approximated by assuming a single-layered leaky artesian aquifer of transmissivity k_1H_1, overlain by a semi-pervious layer of resistance c

$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} \quad \text{or}$$

$$\frac{1}{c} = \frac{1}{(400)10^6} + \frac{1}{(50)10^6} = \frac{9}{(400)10^6}\ , \quad c = (44.4)10^6 \text{ sec}$$

$$\lambda = \sqrt{k_1H_1c} = \sqrt{(6)10^{-3}(44.4)10^6} = 520 \text{ m}$$

$$s_1' = \frac{Q_o}{2\pi k_1 H_1} \ln \frac{1.123\,\lambda}{r} = 0.742 \ln \frac{580}{r}$$

The difference with the true value

$$s_1 = 0.742 \ln \frac{740}{r}$$ equals in this case

$$s_1 - s_1' = 0.742 \ln \frac{740}{580} = 0.18 \text{ m},$$ negligeable in the immediate vicinity of well.

At greater distances from the well on the other hand, the semi-pervious layer of resistance c_2 becomes unimportant, allowing approximation by one aquifer of transmissivity kH

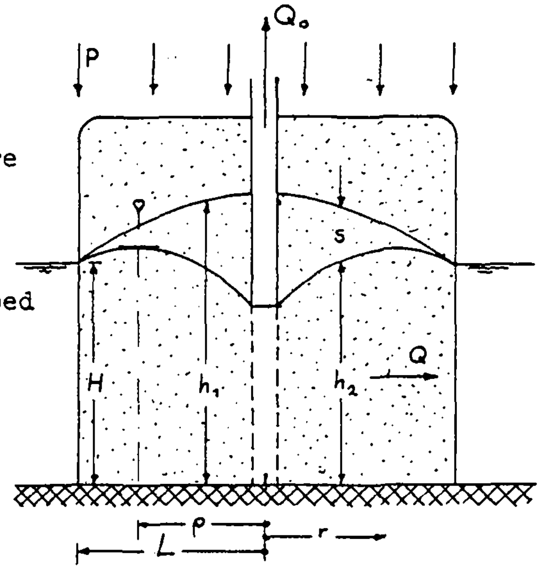
$kH = k_1H_1 + k_2H_2 = (36)10^{-3}$ m^2/sec, overlain by a semi-pervious layer of resistance c_1. This gives

$$\lambda = \sqrt{(36)10^{-3}(400)10^{6}} = 3800 \text{ m}$$ and

$$s_1 = s_2 = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) = 0.124\, K_o\left(\frac{r}{3800}\right).$$

4.11 A circular island has a diameter of 1800 m, is built up of sand and is situated above a horizontal impervious base. The surface water surrounding the island has a constant level of 20.0 m above the base. Due to recharge by rainfall in an amount of $(30)10^{-9}$ m/sec, the groundwater levels inside the island are higher, reaching in the centre to 22.1 m above the base.

In the centre of the island a fully penetrating well with an outside diameter of 0.3 m is constructed. What is the drawdown at the well face and at the water divide when this well is pumped at a constant rate of $(6)10^{-3}$ m^3/sec?



With the notations of the figure at the right, the combined flow pattern due to recharge by rainfall and well abstraction, may be described with

Darcy $$Q = -2\pi r k h_2 \frac{dh_2}{dr}$$

continuity $$Q = \pi r^2 P - Q_o$$

combined $$h_2 dh_2 = -\frac{P}{2k} r dr + \frac{Q_o}{2\pi k}\frac{dr}{r}$$

Integrated between the limits $r = r$, $h_2 = h_2$ and $r = L$, $h_2 = H$

$$H^2 - h_2^2 = -\frac{P}{2k}(L^2 - r^2) + \frac{Q_o}{\pi k}\ln\frac{L}{r}$$

in which the coefficient of permeability k is unknown.

Before pumping, $Q_o = 0$, the formula above gives as water table depth h_1

$$H^2 - h_1^2 = -\frac{P}{2k}(L^2 - r^2)$$

In the centre of the island, $r = 0$, $h_1 = h_{1o}$

$$H^2 - h_{1o}^2 = -\frac{P}{2k}L^2 \quad \text{or} \quad k = \frac{1}{2}\frac{PL^2}{h_{1o}^2 - H^2}$$

Substitution of the data gives

$$k = \frac{1}{2}\,\frac{(30)10^{-9}(900)^2}{(22.1)^2 - (20)^2} = (0.137)10^{-3}\ \text{m/sec}$$

The water table depth before and after pumping thus becomes

$$(20.0)^2 - h_1^2 = -\frac{(30)10^{-9}}{(2)(0.137)10^{-3}}\,\{(900)^2 - r^2\}$$

$$(20.0)^2 - h_2^2 = -\frac{(30)10^{-9}}{(2)(0.137)10^{-3}}\,\{(900)^2 - r^2\} + \frac{(6)10^{-3}}{\pi(0.137)10^{-3}}\ln\frac{900}{r}$$

Simplified

$$400 - h_1^2 = -(88.4)\{1 - (\tfrac{r}{900})^2\}$$

$$400 - h_2^2 = -(88.4)\{1 - (\tfrac{r}{900})^2\} + 13.9\ \ln\frac{900}{r}$$

At the well face, $r = 0.15$ m this gives

$$400 - h_1^2 = -88.4 \qquad ,\ h_1^2 = 488.4\ ,\ h_1 = 22.1$$

$$400 - h_2^2 = -88.4 + 121.0\ ,\ h_2^2 = 367.4\ ,\ h_2 = 19.2$$

$$s = h_1 - h_2 = 2.9\ \text{m}$$

The water divide limits the area over which the recharge by rainfall is abstracted by the well, in formula

$$Q_o = \pi\rho^2 P$$

$(6)10^{-3} = \pi\rho^2(30)10^{-9}$, $\rho = 252$ m , giving as water table depths

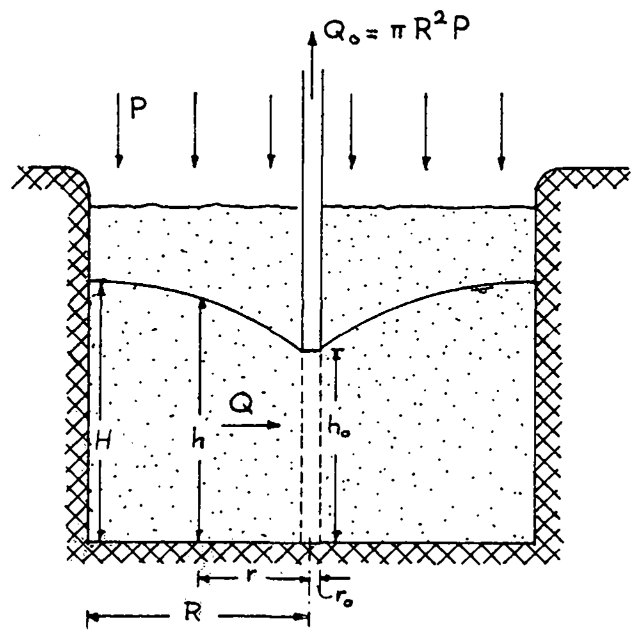
$$400 - h_1^2 = -(88.4)\{1 - (\tfrac{252}{900})^2\} \qquad h_1 = 21.9$$

$$400 - h_2^2 = -(88.4)\{1 - (\tfrac{252}{900})^2\} + 13.9\ \ln\frac{900}{252} \qquad h_2 = 21.5$$

$$s_\rho = h_1 - h_2 = 0.4\ \text{m}$$

4.12 An unconfined aquifer is situated above a horizontal impervious base and is composed of sand with a coefficient of permeability equal to $(0.09)10^{-3}$ m/sec. In plan this aquifer is circular, while the boundary along its circumference is vertical and impervious. The circular basin thus formed has an area of $(2)10^6$ m^2 and is recharged by available rainfall in an amount of $(30)10^{-3}$ m^3/sec. In the centre of the basin a fully penetrating well with an outside diameter of 0.6 m is constructed and pumped at a capacity equal to the full amount of recharge.

What is the groundwater level at the outer circumference of the basin when at the face of the well this level rises to 30 m above the impervious base?



With the notations as indicated in the figure at the right, the equations governing the flow of groundwater in the circular basin become

Darcy $\quad Q = 2\pi rkh \frac{dh}{dr}$

continuity $\quad Q = \pi (R^2 - r^2)P$

combined $\quad hdh = \frac{P}{2k} \frac{R^2 - r^2}{r} dr$

Integration between the limits $r = R$, $h = H$ and $r = r_o$, $h = h_o$ gives

$$H^2 - h_o^2 = \frac{P}{2k} \{2R^2 \ln \frac{R}{r_o} - (R^2 - r_o^2)\}$$

According to the data supplied

$$k = (0.09)10^{-3} \text{ m/sec}, \quad r_o = 0{,}3 \text{ m}, \quad h_o = 30 \text{ m}$$

$$\pi R^2 = (2)10^6 \text{ m}^2 \quad \text{or } R = 798 \text{ m}$$

$\pi R^2 P = (30)10^{-3}$ m^3/sec or $P = (15)10^{-9}$ m/sec. Substituted

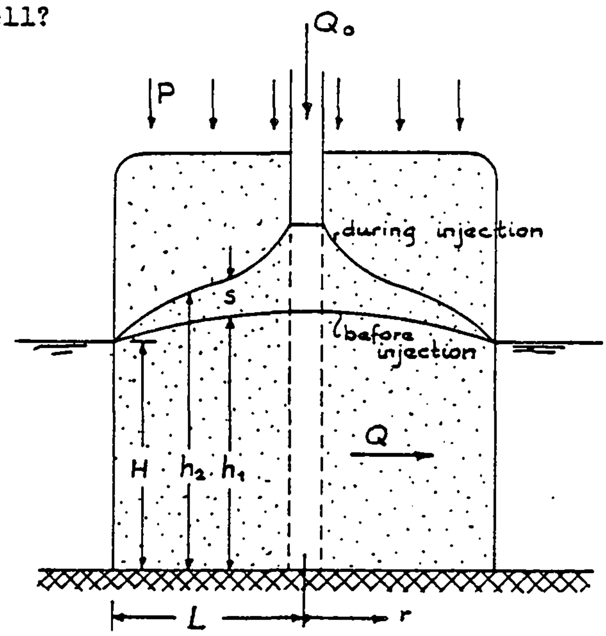
$$H^2 - 900 = \frac{(15)10^{-9}}{(2)(0.09)10^{-3}} \{(2)(798)^2 \ln \frac{798}{0.3} - (798)^2\} \quad \text{or}$$

$$H^2 = 900 + 785 = 1685 \text{ , } H = 41.0 \text{ m}$$

4.13 , A circular island has a diameter of 800 m, is built up of sand with a coefficient of permeability k equal to $(0.35)10^{-3}$ m/sec and is situated above a horizontal impervious base. The surface water surrounding the island has a constant level of 25 m above the base, while the recharge by rainfall amounts to $(50)10^{-9}$ m/sec.

To raise groundwater levels, water is injected in the centre of the island with the help of a fully penetrating well of 0.8 m diameter. The injection rate is kept constant at $28(10^{-3})$ m^3/sec.

What is the rise of the groundwater table at the well face and at a distance of 200 m from the well?



Using the notations of the figure at the right, the combined flow pattern due to recharge by rainfall and injection, can be described with

Darcy $\quad Q = -2\pi r k h_2 \dfrac{dh_2}{dr}$

continuity $\quad Q = Q_o + \pi r^2 P$

combined $\quad h_2 dh_2 = -\dfrac{Q_o}{2\pi k}\dfrac{dr}{r} - \dfrac{P}{2k} r dr$

Integrated between the limits $r = r$, $h_2 = h_2$ and $r = L$, $h_2 = H$

$$h_2^2 - H^2 = \frac{Q_o}{\pi k} \ln \frac{L}{r} + \frac{P}{2k}(L^2 - r^2)$$

Before injection, $Q_o = 0$, this formula gives as water table depth h_1

$$h_1^2 - H^2 = \frac{P}{2k}(L^2 - r^2)$$

At the well face, $r = 0.4$ m, substitution of the data gives

$$h_2^2 - 625 = \frac{(28)10^{-3}}{\pi(0.35)10^{-3}} \ln \frac{400}{0.4} + \frac{(50)10^{-9}}{(2)(0.35)10^{-3}} \{(400)^2 - (0.4)^2\}$$

$$h_2^2 - 625 = 176 + 11.4 \quad , \quad h_2^2 = 812 \quad , \quad h_2 = 28.5$$

$$h_1^2 - 625 = 11.4 \quad , \quad h_1^2 = 636 \quad , \quad \underline{h_1 = 25,2}$$

$$s_o = h_2 - h_1 = 3.3 \text{ m}$$

At a distance of 200 m from the centre

$$h_2^2 - 625 = \frac{(28)10^{-3}}{\pi(0.35)10^{-3}} \ln \frac{400}{200} + \frac{(50)10^{-9}}{2(0.35)10^{-3}} \{(400)^2 - (200)^2\}$$

$$h_2^2 - 625 = 17.7 + 8.6 \quad , \quad h_2^2 = 651.3 \quad , \quad h_2 = 25.5$$

$$h_1^2 - 625 = 8.6 \quad , \quad h_1^2 = 633.6 \quad , \quad \underline{h_1 = 25.2}$$

$$s_{200} = h_2 - h_1 = 0.3 \text{ m}$$

According to the results obtained, the water table variations are only small and the calculation could also have been made assuming a constant coefficient of transmissibility kH

$$s = \frac{Q_o}{2\pi kH} \ln \frac{L}{r} = \frac{(28)10^{-3}}{2\pi(0.35)10^{-3}\,(25)} \ln \frac{400}{r} = 0.510 \ln \frac{400}{r}$$

$r = 0.4$ m $\quad s_o = 0.510 \ln \frac{400}{0.4} = 3.5$ m

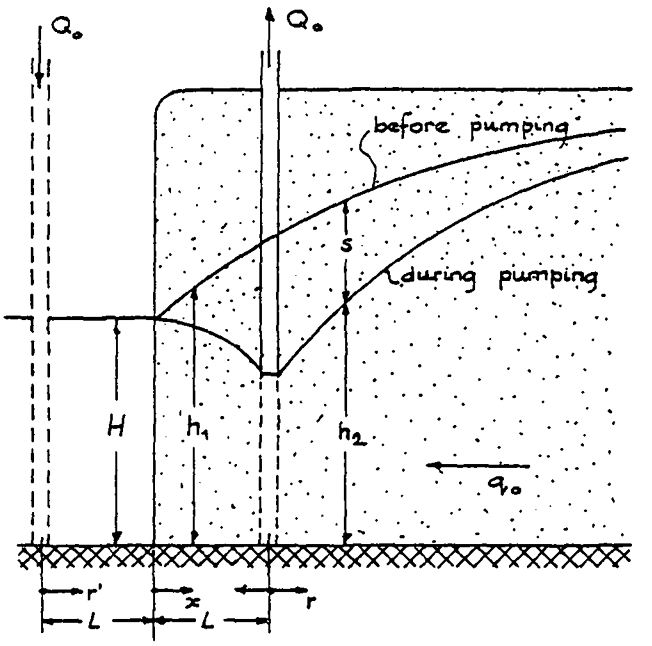
$r = 200$ m, $s_{200} = 0.510 \ln \frac{400}{200} = 0.35$ m

4.14 A semi-infinite unconfined aquifer is situated above an impervious base and bounded at the left by a fully penetrating ditch with a constant and uniform water level, rising to 15 m above the base.

The aquifer has a coefficient of permeability k equal to $(0.40)10^{-3}$ m/sec and discharges groundwater into the ditch in a constant amount of $(75)10^{-6}$ m^3/m'/sec.

At a distance of 150 m from the ditch, a fully penetrating well with an outside diameter of 0.5 m is constructed, to be pumped at a rate of $(12)10^{-3}$ m^3/sec.

What is the drawdown of the water table at the well face and in a point halfway between the well and the ditch?



Before pumping the well, the water table elevation h_1 follows from

$$H^2 - h_1^2 = - \frac{2q_o}{k} x$$

During well pumping, but without the groundwater discharge q_o, the water table elevation may be found by using the method of images

$$H^2 - h_3^2 = \frac{Q_o}{\pi k} \ln \frac{r'}{r}$$ with r' as distance between the point

of observation and the centre of the imaginary recharge well. By superposition the water table depth during pumping and outflow follows at

$$H^2 - h_2^2 = - \frac{2q_o}{k} x + \frac{Q_o}{\pi k} \ln \frac{r'}{r}$$

This gives at the well face with $x = 150$ m, $r = 0.25$ m, $r' = 300$ m

$$225 - h_1^2 = - \frac{(2)(75)10^{-6}}{(0.40)10^{-3}} 150$$

$$225 - h_2^2 = - \frac{(2)(75)10^{-6}}{(0.40)10^{-3}} 150 + \frac{(12)10^{-3}}{\pi(0.40)10^{-3}} \ln \frac{300}{0.25}$$

Simplified

$$225 - h_1^2 = - 56.3 \quad , \quad h_1^2 = 281 \quad , \quad h_1 = 16.8$$

$$225 - h_2^2 = - 56.3 + 67.7 \quad , \quad h_2^2 = 214 \quad , \quad \underline{h_2 = 14.6}$$

$$s_o = h_1 - h_2 = 2.2 \text{ m}$$

Halfway between the well and the ditch, the coordinates are

$x = 75$ m , $r = 75$ m , $r' = 225$ m. Substituted

$$225 - h_1^2 = - \frac{(2)(75)10^{-6}}{(0.40)10^{-3}} 75$$

$$225 - h_2^2 = - \frac{(2)(75)10^{-6}}{(0.40)10^{-3}} 75 + \frac{(12)10^{-3}}{\pi(0.40)10^{-3}} \ln \frac{225}{75} \quad \text{or}$$

$$225 - h_1^2 = - 28.1 \quad , \quad h_1^2 = 253 \quad , \quad h_1 = 15.9$$

$$225 - h_2^2 = - 28.1 + 10.5 \quad , \quad h_2^2 = 243 \quad , \quad \underline{h_2 = 15.6}$$

$$s_{75} = h_1 - h_2 = 0.3 \text{ m}$$

With a constant coefficient of transmissibility kH, the drawdowns would have been found at

$$s = \frac{Q_o}{2\pi kH} \ln \frac{r'}{r}$$

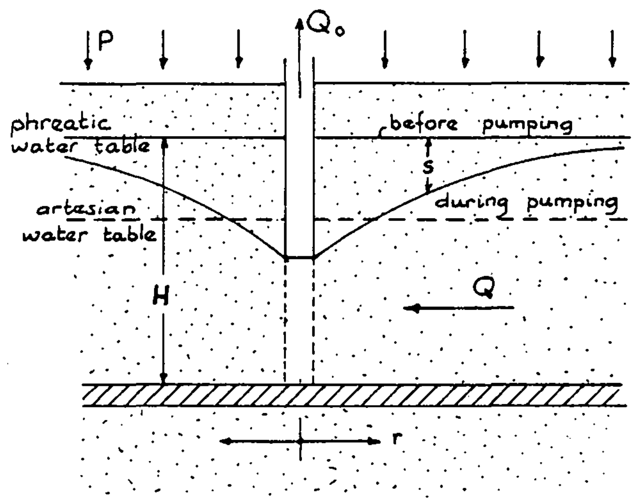
$$s_o = \frac{(12)10^{-3}}{2\pi(0.40)10^{-3}\ (15)} \ln \frac{300}{0.25} = 0.318 \ln 1200 = 2.25 \text{ m}$$

$$s_{75} = 0.318 \ln \frac{225}{75} = 0.318 \ln 3 = 0.35 \text{ m}$$

4.15 An unconfined aquifer of infinite extent is situated above a semi-pervious layer below which artesian water with a constant and uniform water table is present. The coefficient of transmissibility kH of the unconfined aquifer amounts to $(15)10^{-3}$ m^2/sec, its recharge by available rainfall P to $(5)10^{-9}$ m/sec, while the semi-pervious layer has a resistance c of $(25)10^6$ sec against vertical water movement.

In the unconfined aquifer a fully penetrating well with an outside diameter of 0.8 m is constructed. From this well groundwater is abstracted at a constant rate of $(50)10^{-3}$ m^3/sec.

What is the drawdown of the phreatic water table at the well face and at a distance of 250 m from the well? How much artesian water percolates upward in an area with a radius of 250 m around the well?



The drawdown due to pumping a well in an unconfined aquifer above a semi-pervious base equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right)$$

with $\lambda = \sqrt{kHc}$

When λ is small, the drawdown may be approximated by

$$s = \frac{Q_o}{2\pi kH} \ln \frac{1.123\lambda}{r}$$

In the case under consideration

$$\lambda = \sqrt{(15)10^{-3}\ (25)10^{6}} = 613 \text{ m}$$

giving as drawdown at the well face (r = 0.4 m) and at r = 250 m from the well

$$s_o = \frac{(50)10^{-3}}{2\pi(15)10^{-3}} \ln \frac{(1.123)613}{0.4} = (0.53) \ln 1720 = 3.95 \text{ m}$$

$$s_{250} = (0.53)K_o(\frac{250}{613}) = 0.53\ K_o(0.408) = (0.53)(1.097) = 0.58\ m$$

According to the water balance for the area with a radius of 250 m around the well, the abstraction $Q_o = (50)10^{-3}\ m^3/sec$ is composed of 3 parts:

rainfall

$$Q_r = \pi r^2 P = \pi (250)^2 (5) 10^{-9} = (1.0)10^{-3}$$

lateral inflow

$$Q_r = Q_o\ \frac{r}{\lambda} K_1(\frac{r}{\lambda}) = (50)10^{-3}\ \frac{250}{613} K_1\ (\frac{250}{613}) = (20.4)10^{-3}\ K_1(0.407)$$

$$Q_r = (20.4)10^{-3}\ (2.133) \qquad = (43.5)10^{-3}$$

upward percolation $\quad u$

together $\quad u + (44.5)10^{-3}$

from which follows $\quad u = (50)10^{-3} - (44.5)10^{-3}$ or

$$u = (5.5)10^{-3}\ m^3/sec$$

4.16 An unconfined aquifer without recharge from above or from below has a coefficient of transmissibility kH equal to $(3)10^{-3}$ m^2/sec. In this aquifer a strip of land with a width of 500 m is bounded at the left by a fully penetrating ditch and at the right by an impervious dyke

At a distance of 200 m from the ditch a fully penetrating well with an outside diameter of 0.5 m is pumped at a constant rate of $(7)10^{-3}$ m^3/sec.

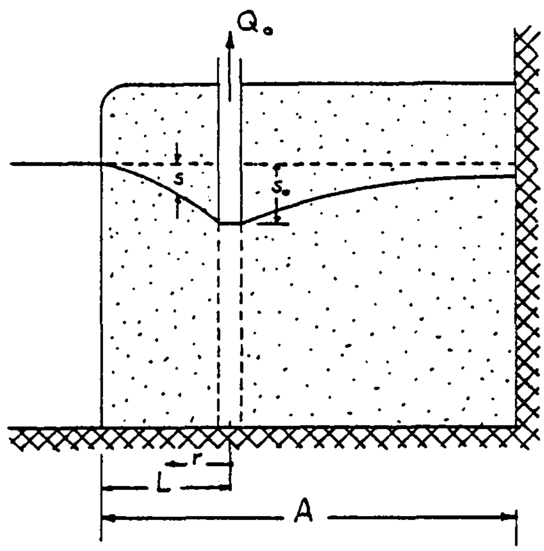
What is the drawdown at the well face and what is roughly the drawdown in a point halfway between the well and the ditch?

For the near vicinity of the well, the integration constant R in the general draw-down formula

$$s = \frac{Q_o}{2\pi kH} \ln \frac{R}{r}$$

has as value

$$R = \frac{4A}{\pi} \operatorname{tg} \frac{\pi L}{2A}$$



In the case under consideration

$$R = \frac{(4)(500)}{\pi} \operatorname{tg} \frac{\pi(200)}{(2)(500)} = \frac{2000}{\pi} \operatorname{tg} \frac{\pi}{5} = 462 \text{ m},$$

giving as drawdown at the well face

$$s_o = \frac{(7)10^{-3}}{2\pi(3)10^{-3}} \ln \frac{462}{0.25} = 0.372 \ln 1848 = 2.80 \text{ m}$$

In a point halfway between the well and the ditch, application of the same formula would give

$$s_{100} = 0.372 \ln \frac{462}{100} = 0.57 \text{ m}$$

This formula, however, needs correction as it would give at the shoreline

$$s_{200} = 0.372 \ln \frac{462}{200} = 0.31 \text{ m, instead of zero.}$$

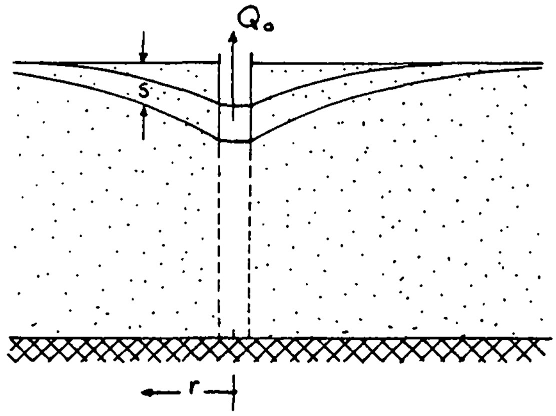
Halfway between the ditch and the well the drawdown will roughly be

$$s_{100} = 0.57 - \frac{0.31}{2} = 0.4 \text{ m}$$

4.21 An unconfined aquifer of infinite extent is situated above an impervious base. The coefficient of transmissibility kH below water table equals $(12)10^{-3}$ m^2/sec, the specific yield μ 17%.

In this aquifer a fully penetrating well with an outside diameter of 0.6 m is set. Starting at t = 0 groundwater is abstracted from this well in an amount of $(40)10^{-3}$ m^3/sec.

What is the drawdown in a point 100 m from the well after 1, 10, 100 and 1000 days of pumping and what is the drawdown at the well face at the latter moment?



The drawdown due to pumping a well in an unconfined aquifer of infinite extent is determined by

$$s = \frac{Q_o}{4\pi kH} W(u^2) \quad \text{with}$$

$$u^2 = \frac{\mu}{4kH} \frac{r^2}{t}$$

Substitution of the data gives for r = 100 m

$$u^2 = \frac{0.17}{(4)(12)10^{-3}} \frac{(100)^2}{t} = \frac{(35.4)10^3}{t}$$

$$s = \frac{(40)10^{-3}}{4\pi(12)10^{-3}} W(u^2) = 0.265\ W(u^2)$$

t =	1	10	100	1000	days
t =	(86.4)	(864)	(8640)	(86400)	$\times 10^3$ sec
u^2 =	0.41	0.041	0.0041	0.00041	
$W(u^2)$ =	0.69	2.66	4.92	7.22	
s =	0.18	0.70	1.31	1.92	m

After 1000 days, the value of u^2 is so small that the drawdown formula may be approximated by

$$s = \frac{Q_o}{4\pi kH} \ln \frac{0.562}{u^2} = \frac{Q_o}{2\pi kH} \ln \frac{0.75}{u}$$

This gives as difference in drawdown for r = 100 m and the well face, r_o = 0.3 m

$$\Delta s = \frac{Q_o}{2\pi kH} \ln \frac{r}{r_o} = (2)(0.265) \ln \frac{100}{0.3} = 3.08 \text{ m}$$

The drawdown at the well face after 1000 days of pumping thus becomes

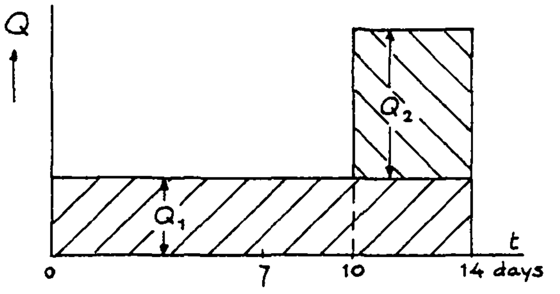
$$s_o = s_{100} + \Delta s = 1.92 + 3.08 = 5.0 \text{ m}$$

4.22 An unconfined aquifer of infinite extent has a coefficient of transmissibility kH equal to $(5)10^{-3}$ m^2/sec, a specific yield μ of 15% and is situated above an impervious base. With a fully penetrating well of 0.5 m diameter groundwater is abstracted from this aquifer for a period of 2 weeks:

during the first 10 days in an amount of 0.01 m^3/sec

during the last 4 days in an amount of 0.03 m^3/sec .

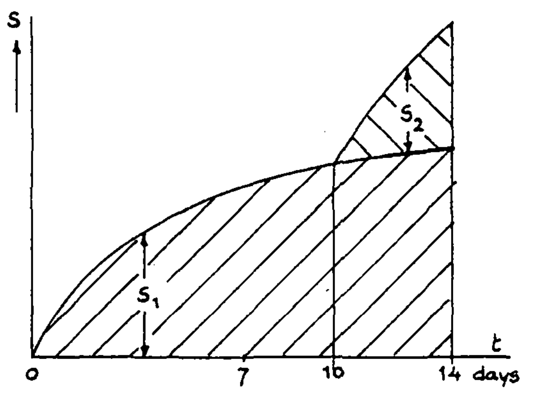
What is the drawdown at the well face at the end of the pumping period?



The drawdown due to pumping a well in an unconfined aquifer of infinite extent is given by

$$s = \frac{Q_o}{4\pi kH} W(u^2) \quad \text{with}$$

$$u^2 = \frac{\mu}{4kH} \frac{r^2}{t}$$



At the well face u^2 is small, allowing to use the approximation

$$s_o = \frac{Q_o}{4\pi kH} \ln \frac{0.562}{u_o^2} \quad \text{or}$$

$$s_o = \frac{Q_o}{2\pi kH} \ln 1.5 \sqrt{\frac{kH}{\mu}} \frac{\sqrt{t}}{r_o}$$

In the case under consideration Q_o is not constant. It may be split, however, in two constant abstractions

$0 < t < 14$ days $\quad Q_1 = 0.01$ m^3/sec

$10 < t < 14$ days $\quad Q_2 = 0.02$ m^3/sec

With the method of superposition the drawdown at t = 14 days thus becomes

$$s_o = \frac{Q_1}{2\pi kH} \ln 1.5 \sqrt{\frac{kH}{\mu}} \frac{\sqrt{t_1}}{r_o} + \frac{Q_2}{2\pi kH} \ln \sqrt{\frac{kH}{\mu}} \frac{\sqrt{t_2}}{r_o} \qquad \text{With}$$

$t_1 = 14$ days $= (1.21)10^6$ sec $\qquad t_2 = 4$ days $= (0.35)10^6$ sec

and the geo-hydrological contents as given

$$s_o = \frac{0.01}{2\pi(5)10^{-3}} \ln 1.5 \sqrt{\frac{(5)10^{-3}}{0.15}} \frac{\sqrt{(1.21)10^6}}{0.25} + \frac{0.02}{2\pi(5)(10^{-3})} \ln 1.5 \sqrt{\frac{(5)10^{-3}}{0.15}} \frac{\sqrt{(0.35)10^6}}{0.25}$$

$$s_o = 0.318 \ln 1200 + 0.637 \ln 650 = 2.26 + 4.13 = 6.4 \text{ m}$$

4.23 An unconfined aquifer of infinite extent is situated above an impervious base. The coefficient of transmissibility kH equals $(8)10^{-3}$ m^2/sec, the specific yield μ 20%. In this aquifer a fully penetrating well with an outside diameter of 0.4 m is pumped for a period of 180 days at a constant rate of $(25)10^{-3}$ m^3/sec.

What is the maximum drawdown at the well face and in a point 800 m from the well?

The unsteady drawdown due to pumping a well in an unconfined aquifer of infinite extent above an impervious base equals

$$s = \frac{Q_o}{4\pi kH} W(u^2) \quad \text{with}$$

$$u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

At the face of the well the drawdown will be maximum at the end of the pumping period or at

$$t = \Delta t = 180 \text{ days} = (15.5)10^6 \text{ sec}$$

$$u^2 = \frac{0.2}{(4)(8)10^{-3}}\frac{(0.2)^2}{(15.5)10^6} = (1.61)10^{-8}$$

$$W(u^2) = 17.37$$

$$s_o = \frac{(25)10^{-3}}{(4)\pi(8)10^{-3}}(17.37) = (0.249)(17.37) = 4.33 \text{ m}$$

At a distance of 200 m from the well, the maximum drawdown may occur at a later moment. When cessation of pumping is materialised by superimposing a recharge of the same magnitude

$$s = \frac{Q_o}{4\pi kH}\{W(u_t^2) - W(u^2_{t-\Delta t})\} \quad \text{or}$$

$$s = 0.249\{W(u_t^2) - W(u^2_{t-\Delta t})\} \quad \text{with}$$

$$u_t^2 = \frac{\mu}{4kH}\frac{r^2}{t} = \frac{0.2}{(4)(8)10^{-3}}\frac{(800)^2}{t} = \frac{(4)10^6}{t}.$$

$$u^2_{t - \Delta t} = \frac{\mu}{4kH}\frac{r^2}{t - \Delta t} = \frac{0.2}{(4)(8)10^{-3}}\frac{800^2}{t - \Delta t} = \frac{(4)10^6}{t - \Delta t}$$

The maximum drawdown occurs at time t determined by

$$\frac{ds}{dt} = \frac{ds}{du^2}\frac{du^2}{dt} = 0$$ The function $W(u^2)$ equals

$$W(u^2) = \int_{u^2}^{\infty} \frac{e^{-u^2}}{u^2}\, du^2$$, giving as requirement

$$\frac{e^{-u_t^2}}{t} = \frac{e^{-u^2_{t - \Delta t}}}{t - \Delta t} \quad \text{or} \quad u^2_{t - \Delta t} - u_t^2 = \ln\frac{t}{t - \Delta t}$$

$$\frac{(4)10^6}{t - (15.5)10^6} - \frac{(4)10^6}{t} = \ln\frac{t}{t - (15.5)10^6}$$ giving by trial and error

$$t = (17)10^6 \text{ sec} = 197 \text{ days and}$$

$$s = 0.249\ \{W(0.235) - W(2.67)\} = 0.249\ \{1.09 - 0.02\} = 0.27 \text{ m}$$

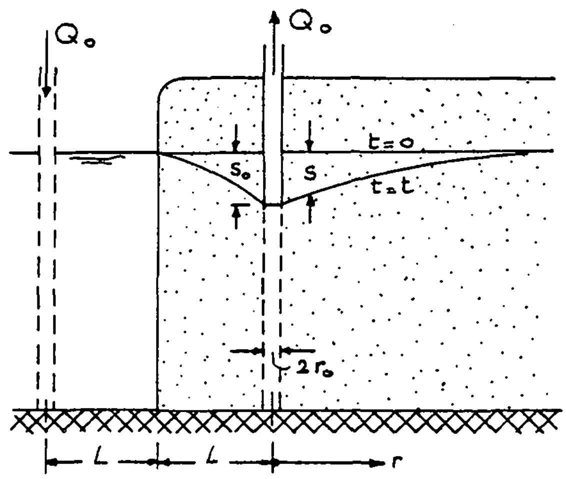
At the end of the pumping period, this drawdown would have been

$$s = 0.249\ W(0.258) = (0.249)(1.02) = 0.25 \text{ m}$$

4.24 A semi-infinite unconfined aquifer is situated above a horinzontal impervious base and bounded by a fully penetrating ditch with a uniform and constant waterlevel of 20 m above the base. The coefficient of transmissibility kH of the aquifer amounts to $(5)10^{-3}$ m^2/sec and the specific yield μ to 30%.

At a distance of 120 m from the ditch a fully penetrating well with an outside diameter of 0.4 m is constructed. Starting at t = 0 this well is pumped at a constant rate of $(8)10^{-3}$ m^3/sec.

What is the drawdown of the groundwater table at the well face after 7 days of pumping and after how many days will 95% of the steady state drawdown here be obtained?



The unsteady flow of groundwater to a well in an aquifer of infinite extent is accompanied by a drawdown s equal to

$$s = \frac{Q_o}{4\pi kH} W(u^2) \quad \text{with}$$

$$u^2 = \frac{\mu}{4kH} \frac{r^2}{t}$$

Using the method of images this gives as drawdown at the face of the well under consideration

$$s_o = \frac{Q_o}{4\pi kH} \{W(u_1^2) - W(u_2^2)\} \quad \text{with}$$

$$u_1^2 = \frac{\mu}{4kH} \frac{r_o^2}{t} \qquad u_2^2 = \frac{\mu}{4kH} \frac{(2L)^2}{t}$$

and as steady state drawdown

$$s_\infty = \frac{Q_o}{2\pi kH} \ln \frac{2L}{r_o}$$

Substituting the data gives

$$s_o = \frac{(8)10^{-3}}{4\pi(5)10^{-3}} \{W(u_1^2) - W(u_2^2)\} = 0.127\{W(u_1^2) - W(u_2^2)\}$$

$$u_1^2 = \frac{0.3}{(4)(5)10^{-3}} \frac{(0.2)^2}{t} = \frac{0.6}{t}$$

$$u_2^2 = \frac{0.3}{(4)(5)10^{-3}} \frac{(240)^2}{t} = \frac{(0.864)10^6}{t}$$

After 7 days = $(0.605)10^6$ sec

$$s_o = 0.127\{W((0.99)10^{-6}) - W(1.43)\} = 0.127(13.25 - 0.11) =$$

$$= 1.67 \text{ m}$$

The steady - state drawdown equals

$$s_\infty = (2)(0.127) \ln \frac{240}{0.2} = 1.80 \text{ m}$$

95% of this value or 1.71 m will be reached at time t determined by

$$1.71 = 0.127\{W(\frac{0.6}{t}) - W\left(\frac{(0.864)10^6}{t}\right)\}$$

from which follows by trial and error

$$t = (1.0)10^6 \text{ sec} = 12 \text{ days}$$

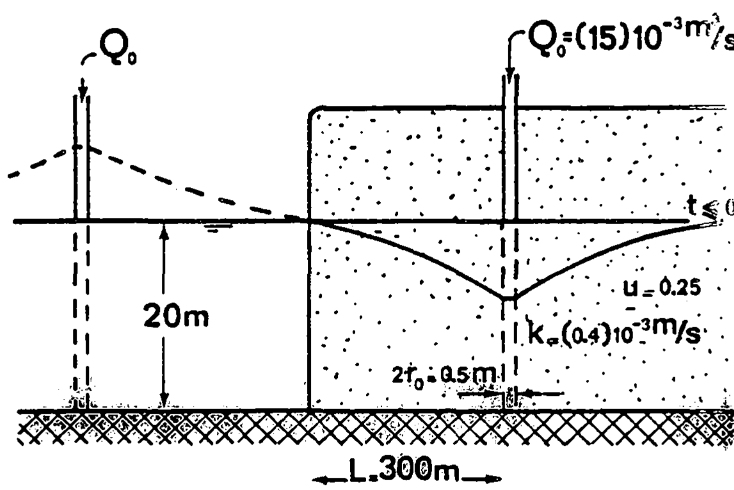
4.25 A semi-infinite unconfined aquifer is composed of sand with a coefficient of permeability k equal to $(0.4)10^{-3}$ m/sec and a specific yield μ of 25%. The aquifer is situated above an impervious base and is bounded by a fully penetrating ditch. Due to absence of recharge, the groundwater table is horizontal, at an elevation of 20 m above the base.

At a distance of 300 m from the ditch a fully penetrating well with an outside diameter of 0.5 m is constructed. Starting at t = 0, the well is pumped at a rate of $(15)10^{-3}$ m^3/sec for a period of 2 days and after that at a constant rate of $(10)10^{-3}$ m^3/sec.

Questions:

a. What is the drawdown of the well face after 5 days of pumping?

b. What is the steady state drawdown at the well face at $Q_o = (10)10^{-3}$ m^3/sec.



Using the method of images, the drawdown at the well face is given by

$$s = \frac{Q_o}{4\pi kH} W(u_1^2) - \frac{Q_o}{4\pi kH} W(u_2^2) \text{ with}$$

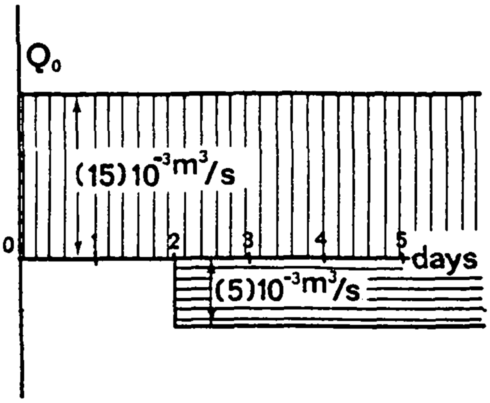
$$u_1^2 = \frac{\mu}{4kH}\frac{r_o^2}{t} = \frac{0.25}{(4)(0.4)10^{-3}(20)}\frac{(0.25)^2}{t} = \frac{0.488}{t}$$

$$u_2^2 = \frac{\mu}{4kH}\frac{(2L)^2}{t} = \frac{0.25}{(4)(0.4)10^{-3}(20)}\frac{(600)^2}{t} = \frac{(2.813)10^6}{t}$$

With u_1^2 small, $W(u_1^2) = \ln \frac{0.562}{u_1^2} = \ln 1.152\ t$, giving as drawdown formula

$$s_o = \frac{Q_o}{4\pi(0.4)10^{-3}(20)}\left[\ln 1.152\ t - W\left\{\frac{(2.813)10^6}{t}\right\}\right] \quad \text{or}$$

$$s_o = 9.95\ Q_o\left[\ln 1.152\ t - W\left\{\frac{(2.813)10^6}{t}\right\}\right]$$



For the abstraction pattern sketched on the right and

$t = 5$ days $= (432)10^3$ sec

$t = (5-2)$days $= (259.2)10^3$ sec

the drawdown at the well face becomes

$$s_o = (9.95)(15)10^{-3}\left[\ln(1.152)(432)10^3 - W\left\{\frac{(2.813)10^6}{(432)10^3}\right\}\right] -$$

$$- (9.95)(5)10^{-3}\left[\ln(1.152)(259.2)10^3 - W\left\{\frac{(2.813)10^6}{(259.2)10^3}\right\}\right]$$

$$s_o = (9.95)(15)10^{-3}\left\{\ln(497.7)10^3 - W(6.51)\right\} -$$

$$- (9.95)(5)10^{-3}\left\{\ln(298.6)10^3 - W(10.85)\right\}$$

$$s_o = (9.95)(15)10^{-3}\,(13.12 - 0.00) - (9.95)(5)10^{-3}\,(12.61 - 0.00)$$

$$s_o = 1.96 - 0.63 = 1.33 \text{ m}$$

The steady-state drawdown at the well face for $Q_o = (10)10^{-3}$ m^3/sec is given by

$$s_o = \frac{Q_o}{2\pi kH}\ln\frac{2L}{r_o} = \frac{(10)10^{-3}}{2\pi(0.4)10^{-3}(20)}\ln\frac{(2)(300)}{0.25} = 1.55 \text{ m}$$

4.26 A circular island with a radius L of 800 m is situated above an impervious base and is composed of sand with a coefficient of transmissibility kH of $(12)10^{-3}$ m^2/sec and a specific yield μ of 25%. In the centre of the island a fully penetrating well with an outside diameter $2r_o$ of 0.6 m is constructed. Starting at t = 0 the well is pumped at a constant rate of $(50)10^{-3}$ m/sec.

After how much time will the drawdown at the well face reach 90% of the steady state value?

Under steady flow conditions, the drawdown at the face of a well in the centre of a circular island is given by

$$s_o = \frac{Q_o}{2\pi kH} \ln \frac{L}{r_o}$$ In the case under consideration

$$s_o = \frac{(50)10^{-3}}{2\pi(12)10^{-3}} \ln \frac{800}{0.3} = 0.6631 \ln 2667 = 5.23 \text{ m and}$$

$$0.9\, s_o = (0.9)(5.23) = 4.71 \text{ m}$$

The unsteady flow to a well in an aquifer of infinite extend is described by

$$s_o' = \frac{Q_o}{4\pi kH} W(u^2) \text{ with } u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

At the well face, this may be simplified to

$$s_o' = \frac{Q_o}{4\pi kH} \ln \frac{0.562}{u_2^2} \text{ with } u_2^2 = \frac{\mu}{4kH}\frac{r_o^2}{t}$$

To keep the drawdown at the outer circumference of the island zero, an infinite number of recharge wells with a combined capacity Q_o will be assumed here. This reduces the drawdown at the well face to

$$s_o' = \frac{Q_o}{4\pi kH}\left\{\ln \frac{0.562}{u_1^2} - W(u_2^2)\right\} \text{ with } u_2^2 = \frac{\mu}{4kH}\frac{L^2}{t}$$

With the date under consideration

$$s'_o = 0.3316 \left\{ \ln 1.20\ t - W\left(\frac{10^7}{3\ t}\right) \right\} \quad \text{and}$$

$$
\begin{aligned}
t &= 10^5 & s'_o &= 0.3316\ (11.695 - 0.000) = 3.88\ \text{m} \\
& 10^6 & &= 0.3316\ (13.998 - 0.009) = 4.64\ \text{m} \\
& 10^7 & &= 0.3316\ (16.300 - 0.830) = 5.13\ \text{m} \\
& (1.1)10^6 & &= 0.3316\ (14.093 - 0.013) = 4.67\ \text{m} \\
& (1.2)10^6 & &= 0.3316\ (14.180 - 0.017) = 4.70\ \text{m}
\end{aligned}
$$

By interpolation it may be inferred that a drawdown of 4.71 m is reached after

$$t = (1.21)10^6 \text{ sec} = 14 \text{ days}$$

4.27 An unconfined aquifer of infinite extent is situated above an impervious base. Its transmissibility kH amount to 0.015 m^2/sec, its specific yield equals 30%. In this aquifer a well with an outside diameter of 0.5 m is set, its screen with a length of 20 m extending over the lower half of the aquifer depth. During the summer half-year water is abstracted from this well in an amount Q_o = (0.05) m^3/sec, while during the winter half-year the same amount of water is recharged.

What is the ultimate variation of the ground-water table at the well face?

The unsteady flow of groundwater to a single well can be described by

$$s = \frac{Q_o}{4\pi kH} W(u^2) \text{ with } u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

At the well face u^2 is always small, giving as good approximation for a fully penetrating well

$$s_o = \frac{Q_o}{4\pi kH} \ln \frac{0.562}{u^2} = \frac{0.05}{4\pi(0.015)} \ln \frac{(0.562)(4)(0.015)}{(0.3)(0.25)^2} t \text{ or}$$

$$s_o = 0.265 \ln 1.80\, t$$

The maximum drawdown at the well face occurs at the end of the pumping period. When pumping starts at t = o, this gives

$t = \frac{1}{2}$ year = $(15.8)10^6$ sec

$$s_o = 0.265 \ln(1.80)(15.8)10^6 = 0.265 \ln(28.4)10^6 = 4.55 \text{ m}$$

The first transition from pumping to recharge is mathematecally accomplished by superimposing a recharge of double capacity and the next transition from recharge to pumping by superimposing an abstraction 2 Q_o (see diagram). This gives

$t = 1\frac{1}{2}$ year

$$s_o = 0.265 \left\{ \ln(3)(28.4)10^6 - 2 \ln(2)(28.4)10^6 + 2 \ln(28.4)10^6 \right\}$$

$$s_o = 0.265 \ln \frac{(3)(1)^2(28.4)^3 10^{18}}{(2)^2(28.4)^2 10^{12}} = 0.265 \ln(\tfrac{3}{4})(28.4)10^6 = 4.47 \text{ m}$$

pumping	recharge	pumping	recharge	pumping	recharge	pumping
$+ Q_o$	$+ Q_o$	$+ Q_o$	$+ Q_o$	$+ Q_o$	$+ Q_o$	$+ Q_o$
	$-2\ Q_o$	$-2\ Q_o$	$-2\ Q_o$	$-2\ Q_o$	$-2\ Q_o$	$-2\ Q_o$
		$+2\ Q_o$	$+2\ Q_o$	$+2\ Q_o$	$+2\ Q_o$	$+2\ Q_o$
			$-2\ Q_o$	$-2\ Q_o$	$-2\ Q_o$	$-2\ Q_o$
				$+2\ Q_o$	$+2\ Q_o$	$+2\ Q_o$
					$-2\ Q_o$	$-2\ Q_o$
						$+2\ Q_o$

t = o ½ 1½ 2½ 3½ year

t = 2½ year

$$s_o = 0.265 \ln \frac{(5)(3)^2(1)^2}{(4)^2(2)^2} (28.4)10^6 = 0.265 \ln \frac{45}{64} (28.4)10^6 = 4.45 \text{ m}$$

t = 3½ year

$$s_o = 0.265 \ln \frac{(7)(5)^2(3)^2(1)^2}{(6)^2(4)^2(2)^2} (28.4)10^6 = 0.265 \ln \frac{1575}{2304} (28.4)10^6 = 4.45 \text{ m}$$

and ultimately

$$s_o = 4.43 \text{ m}$$

Due to partial penetration, an additional drawdown will occur

$$\Delta s_o = \frac{Q_o}{2\pi kH} \frac{1-p}{p} \ln \frac{(1-p)h}{r_o} \quad , \text{ with } p = 0.5$$

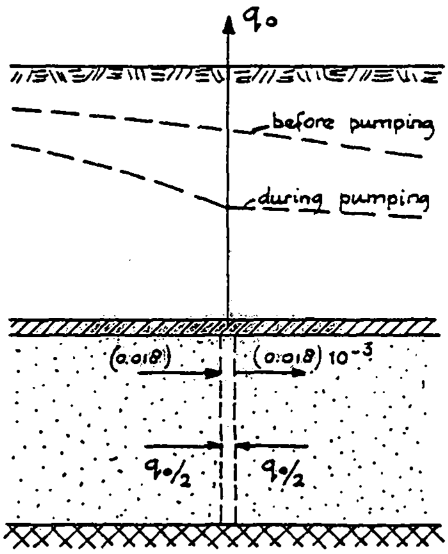
$$\Delta s_o = \frac{(0.05)10^{-3}}{2\pi(0.015)} \frac{0.5}{0.5} \ln \frac{(0.5)(20)}{0.25} = 0.531 \ln 40 = 1.96 \text{ m. This gives}$$

$s_o + \Delta s_o = 4.43 + 1.96 = 6.39$ m and a variation in water table elevation of $(2)(6.39) = 12.8$ m

5.01 A leaky artesian aquifer is situated above an impervious base and overlain by a semi-pervious layer. Above the latter layer an unconfined aquifer is present, the water table of which is maintained at a constant and uniform level. The thickness H of the artesian aquifer equals 50 m, its coefficient of permeability k $(0.36)10^{-3}$ m/sec, while the resistance c of the overlying layer against vertical water movement amounts to $(190)10^{6}$ sec.

The western half of this aquifer is used as catchment area for a water supply company. To its regret this company notes that across the eastern boundary, with a length of 3000 m, artesian water in an amount of $(0.018)10^{-3}$ $m^3/m'/sec$ is flowing out. To intercept this outflow it is decided to construct a line of wells along the boundary. The wells are drilled at intervals b of 150 m, with a diameter of 0.30 m and a screen length of 15 m, from 10 to 25 m below the top of the aquifer.

What is the maximum drawdown of the artesian water table at the face of a well, when all wells are pumped at the same rate Q_o, of such a magnitude that the outflow is reduced to zero.



To solve this problem, the line of wells is first replaced by a gallery with the same capacity per lineal meter

$$q_o = \frac{Q_o}{b}$$

When the gallery operates, the picture at the right gives as remaining outflow q_1

$$q_1 = (0.018)10^{-3} - \frac{q_o}{2}$$

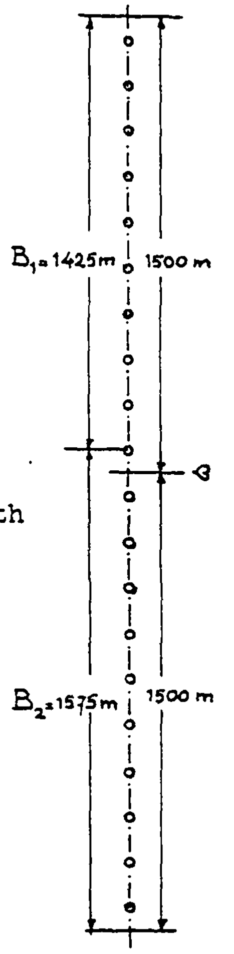
To reduce q_1 to zero clearly asks for a gallery capacity

$$q_o = (0.036)10^{-3} \quad m^3/m'/sec$$

The drawdown due to pumping a fully penetrating gallery of infinite length in a leaky artesian aquifer is given by

$$s' = \frac{q_o}{2}\frac{\lambda}{kH}e^{-x/\lambda} \qquad \text{with}$$

$$\lambda = \sqrt{kHc} = \sqrt{(0.36)10^{-3}(50)(190)10^{6}} = 1850 \text{ m}$$



The drawdown at the face of the gallery thus becomes

$$s'_o = \frac{(0.036)10^{-3}}{2}\frac{1850}{(18)10^{-3}}e^{o} = 1.85 \text{ m}$$

In reality, however, the gallery has only a finite length of 3000 m, by which the drawdown will be smaller

$$s''_o = \beta s'_o \qquad \text{with}$$

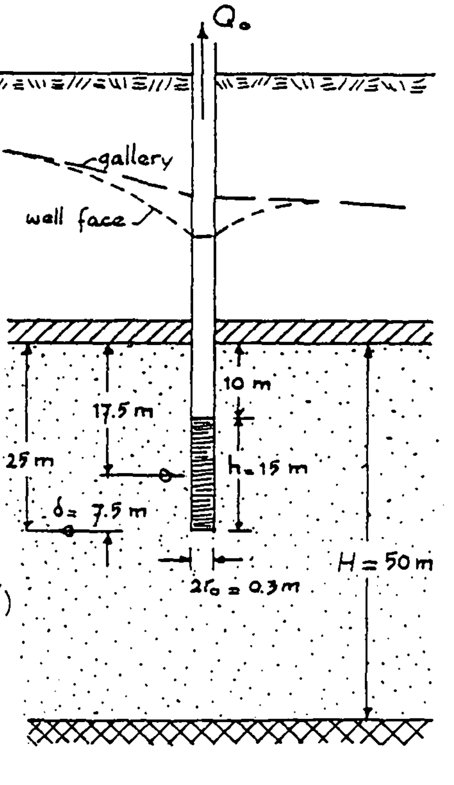
$$\beta = \frac{1}{2}F_1\left(\frac{B_1}{\lambda}\right) + \frac{1}{2}F_1\left(\frac{B_2}{\lambda}\right)$$

The reduction factor β has its largest value in the centre of the line of wells

$$\beta = \frac{1}{2}F_1\left(\frac{1425}{1850}\right) + \frac{1}{2}F_1\left(\frac{1575}{1850}\right) = \frac{1}{2}F_1(0.771) + \frac{1}{2}F_1(0.851) \text{ or}$$

$$\beta = \frac{1}{2}(0.718 + 0.746) = 0.732$$

$$s''_o = (0.732)(1.85) = 1.35 \text{ m}$$



At the well face the drawdown is larger due to point abstraction and partial penetration

$$\Delta s_{p.a.} = \frac{Q_o}{2\pi kH}\ln\frac{b}{2\pi r_o}, \quad Q_o = b.q_o$$

$$\Delta s_{p.a.} = \frac{(150)(0.036)10^{-3}}{2\pi\ (18)10^{-3}}\ln\frac{150}{2\pi(0.15)}$$

$$= (0.0478)\ln 159 = 0.24 \text{ m}$$

$$\Delta s_{pp} = \frac{Q_o}{2\pi kH} \frac{1-p}{p} \ln \frac{\alpha h}{r_o}$$ with α a function of the amount

of penetration p and the amount of eccentricity e

$$p = \frac{h}{H} = \frac{15}{50} = 0.30, \quad e = \frac{\delta}{H} = \frac{7.5}{50} = 0.15, \; \alpha = 0.39$$

$$\Delta s_{pp} = (0.0478) \frac{0.7}{0.3} \ln \frac{(0.39)(15)}{0.15} = 0.41 \text{ m}$$

The maximum drawdown at the well face thus becomes

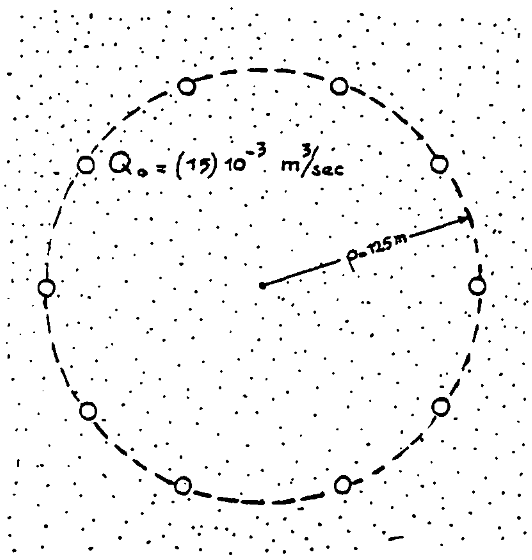
$$s_o = \beta s'_o + \Delta s_{pa} + \Delta s_{pp} = 1.35 + 0.24 + 0.41 = 2.0 \text{ m}$$

5.02 A leaky artesian aquifer is situated between an impervious layer at the bottom and a semi-pervious layer at the top. Above the latter layer phreatic water with a constant and uniform level is present. The thickness H of the leaky artesian aquifer amounts to 60 m, its coefficient of permeability k to $(0.23)10^{-3}$ m/sec, while the overlying semi-pervious layer has a resistance c of $(180)10^{6}$ sec against vertical water movement.

In the leaky artesian aquifer a circular battery of wells is constructed. The battery has a diameter of 250 m and is composed of 10 wells, each with a diameter of 0.4 m and a screen length of 45 m, extending from the top of the aquifer downward. From each well a constant amount of $(15)10^{-3}$ m^3/sec is abstracted.

a. What is the drawdown of the artesian water in the centre of the battery?

b. What is the drawdown of the artesian water table at the well face and halfway between two consecutive wells.



The drawdown due to pumping a single well in a leaky artesian aquifer equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \text{ with } \lambda = \sqrt{kHc}$$

With n wells at an equal distance ρ from the centre of the battery, the drawdown there becomes

$$s_c = \frac{n\, Q_o}{2\pi kH} K_o\left(\frac{\rho}{\lambda}\right)$$

With $\frac{\rho}{\lambda}$ small, less than about 0.2, this formula may be approximated by

$$s_c = \frac{n\, Q_o}{2\pi kH} \ln \frac{1.123\lambda}{\rho}$$

In the case under consideration

$$\rho = 125 \text{ m}, \lambda = \sqrt{(0.23)10^{-3}(60)(180)10^{6}} = 1610 \text{ m and } \frac{\rho}{\lambda} = 0.078,$$

so that the approximation mentioned on page 5.02-a may certainly be applied. The drawdown at the centre of the battery thus becomes

$$s_c = \frac{(10)(15)10^{-3}}{2\pi(0.23)10^{-3}(60)} \ln \frac{(1.123)(1610)}{125} = 1.73 \ln 14.5 = 4.62 \text{ m}$$

The physical meaning of the approximation used above for the calculation of the drawdown in the centre of the battery, is that over the area of the battery the recharge from above is negligeable and the artesian water table horizontal. In the line of wells, the average drawdown is consequently equal to $s_c = 4.62$ m. At the face of each well the drawdown is larger by point abstraction and partial penetration.

$$\Delta s_{p.a} = \frac{Q_o}{2\pi kH} \ln \frac{b}{2\pi r_o}$$ with b as distance between 2 consecutive wells

$$b = \frac{1}{10} \pi(250) = 78.5 \text{ m}$$

$$\Delta s_{p.a} = \frac{1.73}{10} \ln \frac{78.5}{2\pi(0.2)} = 0.173 \ln 62.5 = 0.72 \text{ m}$$

$$\Delta s_{pp} = \frac{Q_o}{2\pi kH} \frac{1-p}{p} \ln \frac{(1-p)h}{r_o}$$ with p as amount of penetration

$$p = \frac{h}{H} = \frac{45}{60} = 0.75$$

$$\Delta s_{pp} = 0.173 \frac{0.25}{0.75} \ln \frac{(0.25)45}{0.2} = 0.058 \ln 56.2 = 0.23 \text{ m}$$

Together

$$s_o = s_c + \Delta s_{p.a} + \Delta s_{p.p} = 4.62 + 0.72 + 0.23 = 5.57 \text{ m}$$

Halfway between two wells the drawdown is smaller by an amount

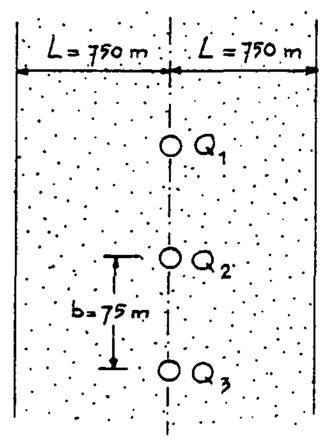
$$\Delta s = \frac{Q_o}{2\pi kH} 0.693 = (0.173)(0.693) = 0.12,$$ together

$$s = s_c - \Delta s = 4.62 - 0.12 = 4.50 \text{ m}$$

5.03 An artesian aquifer without recharge from above or from below is bounded by two fully penetrating ditches, which separate a strip of land of constant width equal to 1500 m. The water levels in both ditches are equal, constant and uniform.

In a line parallel to the ditches and through the centre of the strip of land, three fully penetrating wells with outside diameters of 0.25 m are constructed at equal intervals of 75 m. From the 3 wells together ground-water in an amount of $(50)10^{-3}$ m^3/sec has to be abstracted.

Which subdivision of this total abstraction must be chosen so that the drawdown at the face of each well has the same value?



In the near surrounding of a well in the centre of a strip of land without recharge from above or from below, the drawdown equals

$$s = \frac{Q_o}{2\pi kH} \ln \frac{R}{r} \quad \text{with } R = \frac{4}{\pi} L$$

In the case under consideration, symmetry requires

$Q_1 = Q_3$ giving as drawdowns

$$s_1 = \frac{Q_1}{2\pi kH} \ln \frac{R}{r_o} + \frac{Q_2}{2\pi kH} \ln \frac{R}{b} + \frac{Q_1}{2\pi kH} \ln \frac{R}{2b} = \frac{Q_1}{2\pi kH} \ln \frac{R^2}{2r_o b} + \frac{Q_2}{2\pi kH} \ln \frac{R}{b}$$

$$s_2 = \frac{Q_1}{2\pi kH} \ln \frac{R}{b} + \frac{Q_2}{2\pi kH} \ln \frac{R}{r_o} + \frac{Q_1}{2\pi kH} \ln \frac{R}{b} = \frac{Q_1}{2\pi kH} \ln \frac{R^2}{b^2} + \frac{Q_2}{2\pi kH} \ln \frac{R}{r_o}$$

To satisfy the requirement

$$s_1 = s_2 \quad \text{or} \quad 0 = -s_1 + s_2$$

$$0 = -\frac{Q_1}{2\pi kH} \ln \frac{R^2}{2r_o b} - \frac{Q_2}{2\pi kH} \ln \frac{R}{b} + \frac{Q_1}{2\pi kH} \ln \frac{R^2}{b^2} + \frac{Q_2}{2\pi kH} \ln \frac{R}{r_o}$$

$$0 = -Q_1 \ln \frac{b}{2r_o} + Q_2 \ln \frac{b}{r_o}$$

With the data given

$$0 = -Q_1 \ln \frac{75}{0.25} + Q_2 \ln \frac{75}{0.125} = -5.70\ Q_1 + 6.40\ Q_2$$

$$Q_1 = 1.123\ Q_2$$

With as total capacity

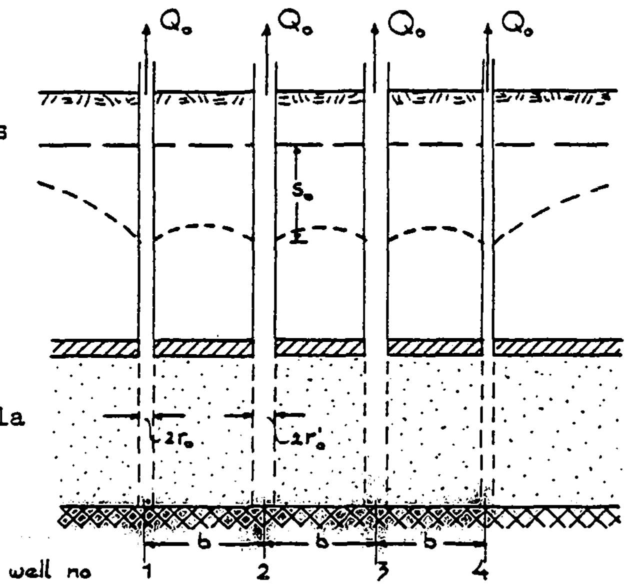
$$(50)10^{-3} = 2Q_1 + Q_2 = 3.246\ Q_2 \quad \text{or}$$

$$Q_2 = (15.4)10^{-3}\ m^3/sec \qquad \text{and} \qquad Q_1 = Q_3 = (17.3)10^{-3}\ m^3/sec$$

5.04 A leaky artesian aquifer is bounded at the bottom by an impervious base and at the top by a semi-pervious layer. Above the semi-pervious layer an unconfined aquifer with a constant and uniform water table is present. The coefficient of transmissibility kH of the artesian aquifer amounts to $(3)10^{-3}$ m^2/sec the resistance c of the overlying layer against vertical water movement to $(50)10^6$ sec.

In the artesian aquifer 4 fully penetrating wells are set on a straight line, at equal intervals b of 200 m and are pumped at the same rate of $(8)10^{-3}$ m^3/sec. The outer wells have a diameter of 0.25 m, the diameter of the inner wells is larger.

How large must the diameter of the inner wells be chosen so that the drawdown at each well face is the same and what is this amount of drawdown?



The drawdown formula for a well in a leaky artesian aquifer reads

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \quad \text{with}$$

$$\lambda = \sqrt{kHc}$$

In case r/λ is small, this formula may be approximated by

$$s = \frac{Q_o}{2\pi kH} \ln \frac{1.123\lambda}{r}$$

Using the method of superposition this gives as drawdown at the faces of the outer and inner wells

$$s_{01} = \frac{Q_o}{2\pi kH} \left\{\ln \frac{1.123\lambda}{r_o} + K_o\left(\frac{b}{\lambda}\right) + K_o\left(\frac{2b}{\lambda}\right) + K_o\left(\frac{3b}{\lambda}\right)\right\}$$

$$s_{02} = \frac{Q_o}{2\pi kH} \left\{K_o\left(\frac{b}{\lambda}\right) + \ln \frac{1.123\lambda}{r_o'} + K_o\left(\frac{b}{\lambda}\right) + K_o\left(\frac{2b}{\lambda}\right)\right\}$$

Equality of both drawdowns requires

$$\ln \frac{1.123\lambda}{r_o} + K_o\left(\frac{3b}{\lambda}\right) = \ln \frac{1.123\lambda}{r_o'} + K_o\left(\frac{b}{\lambda}\right)$$

$$\ln \frac{r_o'}{r_o} = K_o(\frac{b}{\lambda}) - K_o(\frac{3b}{\lambda})$$

With b = 200 m and

$$\lambda = \sqrt{kHc} = \sqrt{(3)10^{-3}(50)10^{6}} = 387 \text{ m}$$

$$\ln \frac{r_o'}{r_o} = K_o(0.517) - K_o(1.55) = 0.897 - 0.200 = 0.697 = \ln 2.01$$

$$r_o' = (2.01)(0.25) = 0.50 \text{ m}$$

The drawdown itself becomes

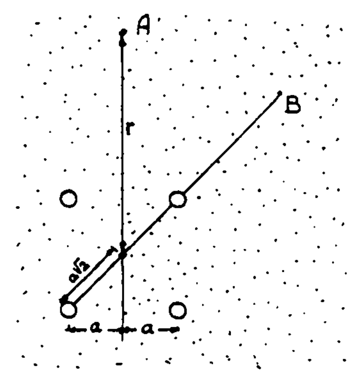
$$s_o = \frac{(8)10^{-3}}{2\pi(3)10^{-3}} \{\ln 3480 + K_o(0.517) + K_o(1.034) + K_o(1.55)\}$$

$$s_o = 0.425 \{8.15 + 0.90 + 0.40 + 0.20\} = 4.1 \text{ m}$$

5.05 A leaky artesian aquifer of infinite extent has a thickness H of 60 m and a coefficient of permeability k of $(0.3)10^{-3}$ m/sec. The aquifer is bounded at the bottom by an impervious base and at the top by a semi-pervious layer with a resistance of $(0.15)10^{9}$ sec against vertical watermovement. Above this layer phreatic water with a constant and uniform level is present.

In the leaky artesian aquifer 4 wells are constructed, at the corners of a square with sides of 60 m. The wells have outside diameters of 0.4 m, with the screens extending from the top of the aquifer 20 m downward. All wells are pumped at a constant rate Q_o of $(20)10^{-3}$ m^3/sec each.

What is the drawdown at the well face and what is the drawdown at a distance of 100 m from the centre of the square?



The drawdown due to pumping a fully penetrating well in a leaky artesian aquifer of infinite extent equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \quad \text{with}$$

$$\lambda = \sqrt{kHc} = \sqrt{(0.3)10^{-3}(60)(0.15)10^{9}} = 1640 \text{ m}$$

When $\frac{r}{\lambda}$ is small, less than 0.16, a 99% accurate approximation may be had with

$$s = \frac{Q_o}{2\pi kH} \ln \frac{1.123\lambda}{r} = \frac{(20)10^{-3}}{2\pi(0.3)10^{-3}(60)} \ln \frac{(1.123)(1640)}{r} \quad \text{or}$$

$$s = 0.177 \ln \frac{1850}{r}$$

Using the method of superposition, the drawdown in a point at distances r_1, r_2, r_3 and r_4 from the 4 wells now becomes

$$s = 0.177 \ln \frac{(1850)^4}{(r_1)(r_2)(r_3)(r_4)}$$

At the well face the distances r_1 to r_4 equal

$$r_1 = r_o = 0.2 \text{ m}, \quad r_2 = r_3 = 2a = 60 \text{ m}, \quad r_4 = 2a\sqrt{2} = 85 \text{ m}$$

$$s_o = 0.177 \ln \frac{(1850)^4}{(0.2)(60)^2(85)} = 0.177 \ln (1.92)10^8 = 3.37 \text{ m}$$

At the well face in the meanwhile, the influence of partial penetration must still be considered. This results in an additional drawdown

$$\Delta s_o = \frac{Q_o}{2\pi kH} \frac{1 - p}{p} \ln \frac{(1 - p)h}{r_o} \quad \text{with h as screen length and}$$

$$p = \frac{h}{H} = \frac{20}{60} = 0.333$$

$$\Delta s_o = 0.177 \frac{0.666}{0.333} \ln \frac{(0.666)(20)}{0.2} = 0.354 \ln 66.7 = 1.51 \text{ m}$$

The drawdown at the face of the partially penetrating well thus becomes

$$s'_o = s_o + \Delta s_o = 3.37 + 1.51 = 4.9 \text{ m}$$

At a distance of 100 m from the centre of the group of wells, the drawdown will vary. Extreme values will occur in the points A and B of the accompanying sketch. With

$$r_A : \sqrt{(r - a)^2 + a^2} = 76 \text{ m} \qquad \text{and } \sqrt{(r + a)^2 + a^2} = 133 \text{ m}$$

$$r_B : r - a\sqrt{2} = 58, \qquad r + a\sqrt{2} = 142 \qquad \text{and} \qquad \sqrt{r^2 + (a\sqrt{2})^2} = 109 \text{ m}$$

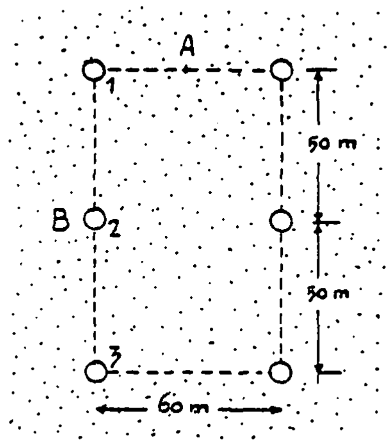
the drawdowns will be

$$S_A = 0.177 \ln \frac{(1850)^4}{(76)^2(133)^2} = 0.177 \ln (1.15)10^4 = 1.09 \text{ m}$$

$$S_B = 0.177 \ln \frac{(1850)^4}{(58)(142)(109)^2} = 0.177 \ln (1.20)10^4 = 1.10 \text{ m}$$

5.06 A leaky artesian aquifer has a coefficient of transmissibility kH equal to $(5)10^{-3}$ m^2/sec and is bounded at the bottom by an impervious base and at the top by a semi-pervious layer with a resistance c of $(20)10^6$ sec against vertical water movement. Above this semi-pervious layer phreatic water with a constant and uniform level is present.

In the leaky artesian aquifer 6 fully penetrating wells with outside diameters of 0.5 m are constructed and pumped at equal capacities Q_o. The wells are set along the circumference of a rectangle as shown in the picture at the right.



What must be the well capacity Q_o so that over the full area of the rectangle the lowering of the artesian water table is at least 3 m and what is the maximum drawdown at the well face?

The minimum drawdown will occur in point A, having as distances to the well centres

$$r_1 = 30 \text{ m}$$

$$r_2 = \sqrt{(30)^2 + (50)^2} = 58.3 \text{ m}$$

$$r_3 = \sqrt{(30)^2 + (100)^2} = 104.4 \text{ m}$$

With $\lambda = \sqrt{kHc} = \sqrt{(5)10^{-3}(20)10^6} = 316$ m, the drawdown due to pumping all wells with the same capacity Q_o equals

$$s_A = \frac{Q_o}{2\pi(5)10^{-3}} \{2\, K_o(\frac{30}{316}) + 2K_o(\frac{58.3}{316}) + 2K_o(\frac{104.4}{316})\}$$

$$s_A = (63.7)\, Q_o \{2.48 + 1.83 + 1.29\} = (356)Q_o$$

Going out from the requirement $s_A = 3$ m, gives

$$Q_o = \frac{3}{356} = (8.4)10^{-3} \text{ m}^3/\text{sec}$$

The maximum drawdown will occur at the face of well B

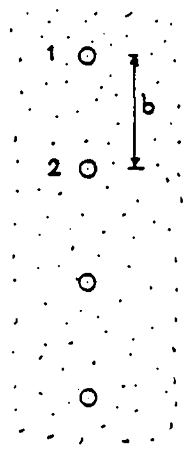
$$s_B = \frac{(8.4)10^{-3}}{2\pi(5)10^{-3}}\{\ln \frac{(1.123)(316)}{0.25} + K_o(\frac{60}{316}) + 2K_o(\frac{50}{316}) + 2K_o(\frac{67.1}{316})\}$$

$$s_B = 0.268\ \{7.26 + 1.80 + 2(1.98) + 2(1.69)\} = 4.4\ m$$

5.07 A leaky artesian aquifer has a thickness of 20 m, a coefficient of permeability of $(0.15)10^{-3}$ m/sec, is bounded at the bottom by an impervious base and at the top by a semi-pervious layer with a resistance of $(50)10^{6}$ sec against vertical water movement. Above this semi-pervious layer an unconfined aquifer with a constant and uniform water table is present.

In the artesian aquifer 4 wells are set on a straight line, at equal intervals of 100 m and are pumped at constant rates of $(8)10^{-3}$ m^3/sec each. The wells have equal diameters of 0.30 m, but the inner wells are fully penetrating, while the outer wells only partially penetrate the saturated thickness of the aquifer.

Calculate the screen length h of the outer wells, extending from the top of the aquifer downward, so that at each well face the drawdown has the same value. How large is this drawdown?



When all wells are fully penetrating, the drawdowns of the outer and inner wells amount to

$$s_1 = \frac{Q_o}{2\pi kH}\{K_o(\frac{r_o}{\lambda}) + K_o(\frac{b}{\lambda}) + K_o(\frac{2b}{\lambda}) + K_o(\frac{3b}{\lambda})\}$$

$$s_2 = \frac{Q_o}{2\pi kH}\{K_o(\frac{r_o}{\lambda}) + 2K_o(\frac{b}{\lambda}) + K_o(\frac{2b}{\lambda})\}$$

with $\lambda = \sqrt{kHc}$ and $K_o(\frac{r_o}{\lambda}) \approx \ln \frac{1.123\lambda}{r_o}$

In the case under consideration

$$\lambda = \sqrt{(0.15)10^{-3}\,(20)(50)10^{6}} = 387 \text{ m}$$

$$s_1 = \frac{(8)10^{-3}}{2\pi(3)10^{-3}}\{\ln\frac{(1.123)(387)}{0.15} + K_o(\frac{100}{387}) + K_o(\frac{200}{387}) + K_o(\frac{300}{387})\}$$

$$s_1 = 0.424\,\{\ln 2900 + K_o(0.259) + K_o(0.518) + K_o(0.777)\}$$

$$s_1 = 0.424\,\{7.963 + 1.509 + 0.895 + 0.586\} = 4.64 \text{ m}$$

$$s_2 = 0.424\,\{7.963 + (2)(1.509) + 0.895\} = 5.03 \text{ m}$$

The difference of 0.39 m must equal the influence of partial penetration

$$\Delta s_o = \frac{Q_o}{2\pi kH} \frac{1 - p}{p} \ln \frac{(1 - p)h}{r_o} \qquad \text{with} \quad h = pH$$

$$0.39 = 0.424 \frac{1 - p}{p} \ln \frac{(1 - p)(p)(20)}{0.15}$$

By trial and error this gives

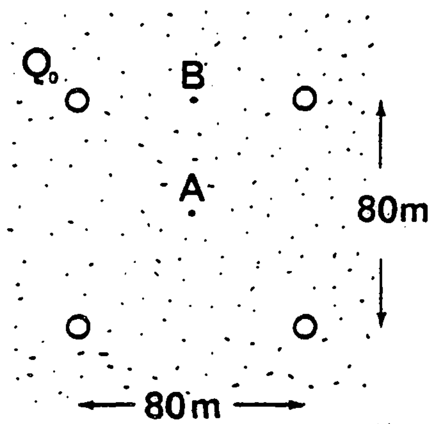
$$p = 0.77 \qquad \text{and} \qquad h = (0.77)(20) = 15.4 \text{ m}$$

5.08 A leaky artesian aquifer has a thickness H of 45 m and is composed of sand with a coefficient of permeability k equal to $(12)10^{-5}$ m/sec. At the bottom the aquifer is bounded by an impervious base and at the top by a semi-pervious layer with a resistance c of $(20)10^{6}$ sec against vertical water movement. Above this semi-pervious layer phreatic water with a constant and uniform level is present. In the leaky artesian aquifer 4 wells are set at the corners of a square with sides of 80 m and are pumped at equal capacities Q_o. The outside diameter of the well screen amounts to 0.4 m, while the screen penetrates the aquifer over a distance of 15 m.

Question:

a. What must be the well capacity Q_o so that over the full area of the square the lowering of the artesian water table is at least 3 m?

b. What is the resulting drawdown at the face of each well?



The drawdown accompanying the flow of water to a well in a leaky artesian aquifer is given by

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \text{ with}$$

$$\lambda = \sqrt{kH\,c} = \sqrt{(12)10^{-5}(45)(20)10^{6}} = 329 \text{ m}$$

The minimum lowering of the groundwater table occurs either in point A or in point B

$$s_A = \frac{Q_o}{2\pi kH} 4K_o\left(\frac{40\sqrt{2}}{329}\right) = \frac{Q_o}{2\pi kH} 4K_o(0.172) = \frac{Q_o}{2\pi kH}(4)(1.898) =$$

$$= 7.59 \frac{Q_o}{2\pi kH}$$

$$s_B = \frac{Q_o}{2\pi kH}\left\{ 2K_o\left(\frac{40}{329}\right) + 2K_o\left(\frac{40\sqrt{5}}{329}\right)\right\} = \frac{Q_o}{2\pi kH}\left\{2K_o(0.122) + 2K_o(0.272)\right\} =$$

$$= \frac{Q_o}{2\pi kH}\left\{(2)(2.232) + (2)(1.463)\right\} = 7.39\,\frac{Q_o}{2\pi kH}$$

The drawdown in point B is the deciding factor, requiring a capacity larger than

$$Q_o = \frac{2\pi kH\ s}{7.39} = \frac{2\pi(12)10^{-5}(45)(3)}{7.39} = (13.8)10^{-3}\ m^3/sec$$

At the face of a fully penetrating well, the drawdown equals

$$s_o = \frac{Q_o}{2\pi kH}\left\{\ln\frac{(1.123)(329)}{0.2} + 2K_o\left(\frac{80}{329}\right) + K_o\left(\frac{80\sqrt{2}}{329}\right)\right\} =$$

$$= \frac{Q_o}{2\pi kH}\left\{\ln 1847 + 2K_o(0.243) + K_o(0.344)\right\}$$

$$= \frac{Q_o}{2\pi kH}\left\{7.52 + (2)(1.568) + (1.248)\right\} = 11.90\,\frac{Q_o}{2\pi kH} \quad \text{or}$$

$$s_o = (11.90)\,\frac{(13.8)10^{-3}}{2\pi(12)10^{-5}(45)} = 4.84\ m$$

Due to partial penetration, the drawdown at the well face will be larger by an amount

$$\Delta s_o = \frac{Q_o}{2\pi kH}\,\frac{1-p}{p}\,\ln\frac{(1-p)h}{r_o} \qquad \text{with}$$

$$p = \frac{h}{H} = \frac{15}{45} = 0.333$$

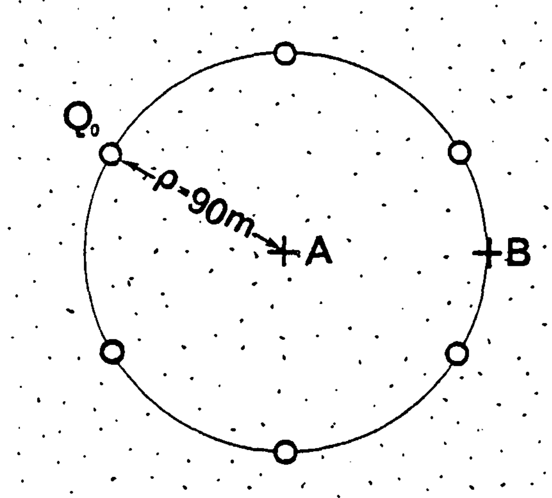
$$\Delta s_o = \frac{(13.8)10^{-3}}{2\pi(12)10^{-5}(45)}\,\frac{1-0.333}{0.333}\,\ln\frac{(1-0.333)15}{0.2} = 3.18\ m \text{ and}$$

$$s_o + \Delta s_o = 4.84 + 3.18 = 8.02\ m$$

5.09 A leaky artesian aquifer is situated between an impervious layer at the bottom and a semi-pervious layer at the top. Above the latter layer phreatic water with a constant and uniform level is present. The thickness H of the leaky artesian aquifer amounts to 50 m, its coefficient of permeability k to $(0.30)10^{-3}$ m/sec, while the overlying semi-pervious layer has a resistance of $(200)10^{6}$ sec against vertical water movement.

In the leaky artesian aquifer a circular battery of wells is constructed. The battery has a diameter of 180 m and is composed of 6 wells with diameters of 0.4 m, set at equal intervals.

At what minimum capacity Q_o must the wells be pumped so that over the full area of the circular battery the lowering of the artesian water table is at least 5 m?



The drawdown due to pumping a well in a leaky artesian aquifer is given by

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \text{ with}$$

$$\lambda = \sqrt{kHc} = \sqrt{(0.3)10^{-3}(50)(200)10^{6}} = 1732 \text{ m}$$

This value is so large, that the Bessel function may be replaced by its logarithmic approximation

$$s = \frac{Q_o}{2\pi kH} \ln \frac{1.123\ \lambda}{r} = \frac{Q_o}{2\pi kH} \ln \frac{1945}{r}$$

At the same time this means that inside the battery of wells the recharge from above is negligible and that everywhere the drawdown is the same, equal to the drawdown at the well centre

$$s_A = \frac{6Q_o}{2\pi kH} \ln \frac{1945}{90} = 18.44 \frac{Q_o}{2\pi kH}$$

Only in the immediate vicinity of the wells do deviations occur and at point B the drawdown is less by an amount

$$s_A - s_B = \frac{Q_o}{2\pi kH}(0.693) \qquad \text{This gives}$$

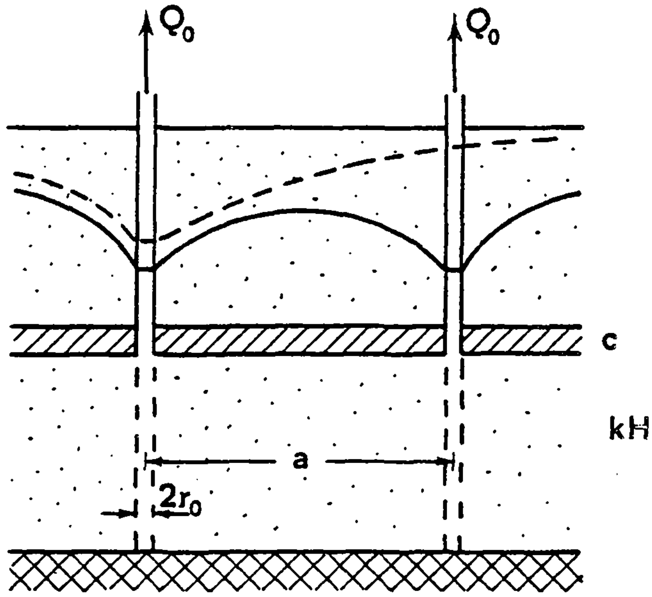
$$s_B = 17.75\,\frac{Q_o}{2\pi kH}$$

from which as minimum value of Q_o follows

$$Q_o = \frac{2\pi kH\ s_B}{17.75} = \frac{2\pi(0.30)10^{-3}(50)(5)}{17.75} = (26.55)10^{-3}\ m^3/sec$$

5.10 A leaky artesian aquifer is situated above an impervious base and is overlain by a semi-pervious layer. Above the latter layer an unconfined aquifer is present, with a water table that is maintained at a constant and uniform level. The coefficient of transmissibility kH of the leaky artesian aquifer amounts to 0.02 m^2/sec, while for a capacity of $(50)10^{-3}$ m^3/sec the drawdown at the face of a fully penetrating well with an outside diameter of 0.6 m equals 3.0 m.

In the leaky artesian aquifer mentioned above, a second well must be constructed of the same design and capacity. What is the minimum distance between the two wells so that the lowering of the artesian water table does not exceed a value of 3.5 m?



For a single well in a leaky artesian aquifer, the drawdown equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right)$$

and at the well face

$$s_o = \frac{Q_o}{2\pi kH} \ln \frac{1.123\ \lambda}{r_o}$$

In the case under consideration

$$3.0 = \frac{(50)10^{-3}}{2\pi(0.02)} \ln \frac{1.123\ \lambda}{0.3} \quad \text{or}$$

$$\ln \frac{1.123\ \lambda}{0.3} = 7.540 = \ln\ 1882 \quad \text{or}$$

$$\lambda = \frac{(1882)(0.3)}{1.123} = 503\ \text{m}$$

With both wells in operation, the drawdown at the well face will be

$$s_o = \frac{Q_o}{2\pi kH} \ln \frac{1.123\ \lambda}{r_o} + \frac{Q_o}{2\pi kH} K_o\left(\frac{a}{\lambda}\right)$$

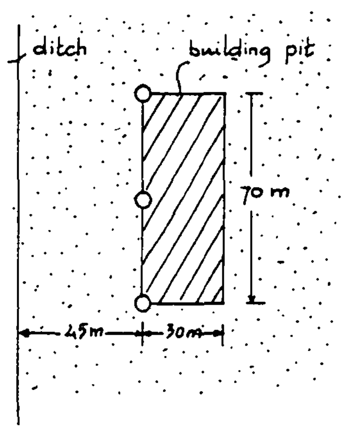
$$3.5 = 3.0 + \frac{(50)10^{-3}}{2\pi(0.02)} K_o\left(\frac{a}{503}\right)$$

$$K_o\left(\frac{a}{503}\right) = \frac{(0.5)(2)\pi(0.02)}{(50)10^{-3}} = 1.2566 = K_o(0.341) \quad \text{or}$$

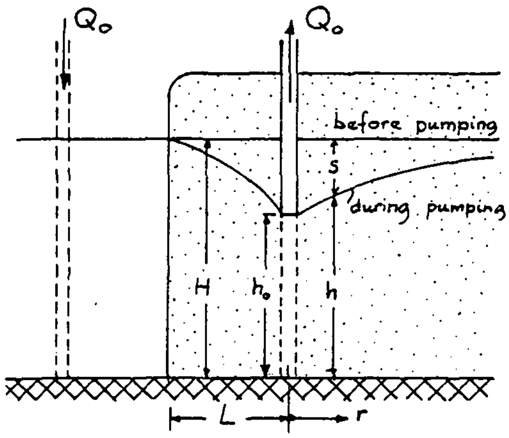
$$a > (503)(0.341) = 172\ \text{m}$$

5.11 An semi-infinite unconfined aquifer has a coefficient of permeability k equal to $(0.17)10^{-3}$ m/sec, is situated above a horizontal impervious base and bounded by a fully penetrating ditch. Rainfall and evapo-transpiration are about the same with as consequence that the ground-water table is horizontal, equal to the constant water level in the bounding ditch at 20 m above the impervious base.

In the unconfined aquifer a building pit must be drained for which purpose 3 fully penetrating wells have been constructed as indicated in the sketch at the right. The wells have equal diameters of 0.3 m and are pumped at the same rate Q_o.

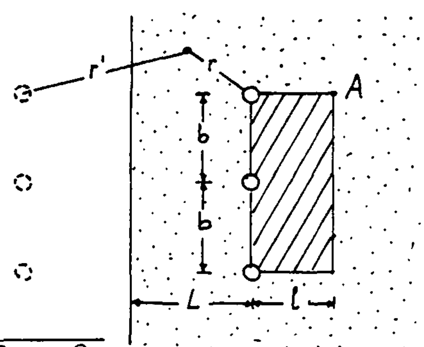


What is the minimum rate of abstraction necessary to lower the groundwater table with at least 2 m over the full area of the building pit. What is the maximum drawdown at the well face and what is the lowering of the groundwater table 1 km from the ditch?



With an unconfined aquifer above an impervious base, the drawdown due to pumping a single well at a distance L from a ditch with constant water table, follows from

$$H^2 - h^2 = \frac{Q_o}{\pi k} \ln \frac{r'}{r}$$



In the case under consideration, the drawdown will be minimum in point A. Using the method of superposition, the remaining water table depth here is given by

$$H^2 - h_A^2 = \frac{Q_o}{\pi k} \{\ln \frac{2L+l}{l} + \ln\sqrt{\frac{(2L+l)^2 + b^2}{l^2 + b^2}} +$$

$$+ \ln\sqrt{\frac{(2L+l)^2 + (2b)^2}{l^2 + (2b)^2}} \}$$

With $H = 20$ m, $s_A = 2$ m, h_A becomes $20 - 2 = 18$ m, giving with the data supplied

$$(20)^2 - (18)^2 = \frac{Q_o}{\pi(0.17)10^{-3}} \ln\left(\frac{120}{30}\right) \sqrt{\frac{(120)^2+(35)^2}{(30)^2+(35)^2}} \sqrt{\frac{(120)^2+(70)^2}{(30)^2+(70)^2}}$$

$$400 - 324 = \frac{Q_o}{\pi(0.17)10^{-3}} \ln \frac{120}{30}\,\frac{125}{46}\,\frac{139}{76} \quad \text{or}$$

$$Q_o = \frac{(76)\pi(0.17)10^{-3}}{2.99} = (13.6)10^{-3} \text{ m}^3/\text{sec}$$

The maximum drawdown occurs at the face of the centre well

$$H^2 - h_o^2 = \frac{Q_o}{\pi k} \ln \frac{2L}{r_o}\,\frac{(2L)^2 + b^2}{b^2}$$

$$(20)^2 - h_o^2 = \frac{(13.6)10^{-3}}{\pi(0.17)10^{-3}} \ln \frac{90}{0.15}\,\frac{(120)^2 + (35)^2}{(35)^2}$$

$$400 - h_o^2 = 25.5 \ln \frac{90}{0.15}\,\frac{15625}{1225} = 25.5 \ln 7653 = 228$$

$$h_o^2 = 400 - 228 = 172,\ h_o = 13.1 \text{ m and}$$

$$s_o = H - h_o = 20 - 13.1 = 6.9 \text{ m}$$

The drawdown $s_o = 6.9$ m in the meanwhile represents the lowering of the water level inside the well when the well losses - if present - are neglected. Outside the wells the water table is higher by the surface of seepage m_o

$$m_o = \frac{H}{2}\left(1 - \frac{h_o}{H}\right)^2 = \frac{20}{2}\left(1 - \frac{15.2}{20}\right)^2 = 0.6 \text{ m}$$

giving as real drawdown at the well face

$$s_o' = s_o - m_o = 6.9 - 0.6 = 6.3 \text{ m}$$

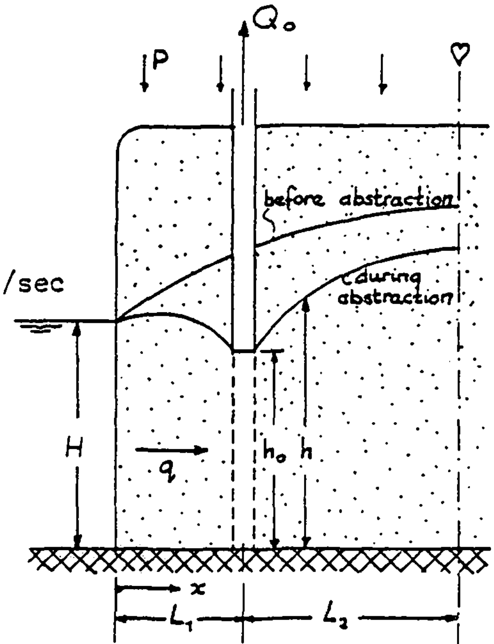
At 1 km = 1000 m from the ditch, the drawdown equals

$$s = \frac{3Q_o}{2\pi kH} \ln \frac{1000 + 45}{1000 - 45} = \frac{(3)(25.5)}{40} \ln 1.095 = 0.17 \text{ m.}$$

5.12 A semi-infinite unconfined aquifer has a coefficient of permeability k equal to $(0.25)10^{-3}$ m/sec, is situated above an impervious base and bounded by a fully penetrating ditch with a constant water level, rising to 12 m above the impervious base. From the aquifer the ditch receives groundwater, originating from available rainfall in an amount of $(30)10^{-9}$ m/sec over an area extending to 1500 m beyond the ditch.

At a distance of 300 m parallel to the ditch a line of 9 fully penetrating wells is constructed. The wells have diameters of 0.4 m, are set at equal intervals b of 110 m and are pumped at a constant rate of $(4.5)10^{-3}$ m^3/sec each.

What is the lowest waterlevel inside the well, neglecting well losses?



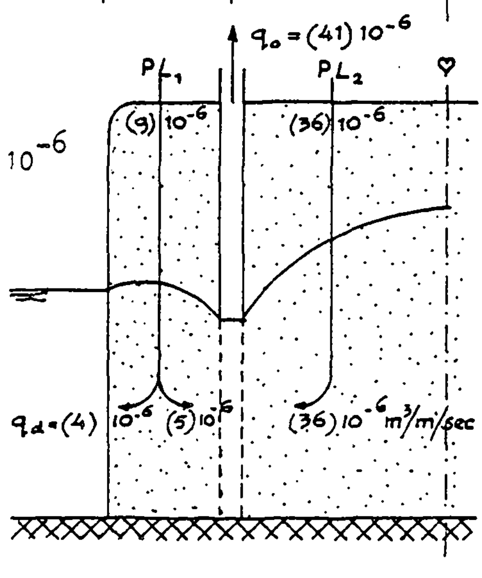
When provisionally the line of wells is replaced by a infinite gallery with the same capacity per unit length

$$q_o = \frac{Q_o}{b} = \frac{(4.5)10^{-3}}{110} = (41)10^{-6} m^3/m'/sec$$

the flow pattern caused by rainfall and abstraction may easily be determined. According to the figure at the right, there remains an outflow of groundwater into the ditch with as magnitude

$$q_d = P(L_1 + L_2) - q_o$$

$$= (30)\ 10^{-9}\ (1500) - (41)10^{-6} = (4)10^{-6}$$



For the strip of land between the ditch and the gallery, the equations of flow thus become

Darcy $\quad q = -kh \frac{dh}{dx}$

Continuity $\quad q = -q_d + Px$

combined $h dh = \frac{q_d}{k} dx - \frac{P}{k} x dx$

Integrated between the limits $x = 0$, $h = H$ and $x = L_1$, $h = h_o$

$$h_o^2 - H^2 = \frac{2q_d}{k} L_1 - \frac{P}{k} (L_1)^2 \quad \text{or with the data given}$$

$$h_o^2 - (12)^2 = \frac{(2)(4)10^{-6}}{(0.25)10^{-3}} (300) - \frac{(30)10^{-9}}{(0.25)10^{-3}} (300)^2$$

$$h_o^2 - 144 = 9.6 - 10.8 , \quad h_o^2 = 142.8, \quad h_o = 11.9 \text{ m}$$

Before pumping the gallery, the full recharge by rainfall flowed out to the ditch

$$q_d' = P(L_1 + L_2) = (30)10^{-9}(1500) = (45)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

giving as watertable elevation h_g at the gallery

$$h_g^2 - H^2 = \frac{2q_d'}{k} L_1 - \frac{P}{k} (L_1)^2$$

$$h_g^2 - 144 = \frac{(2)(45)10^{-6}}{(0.25)10^{-3}} (300) - \frac{(30)10^{-9}}{(0.25)10^{-3}} (300)^2$$

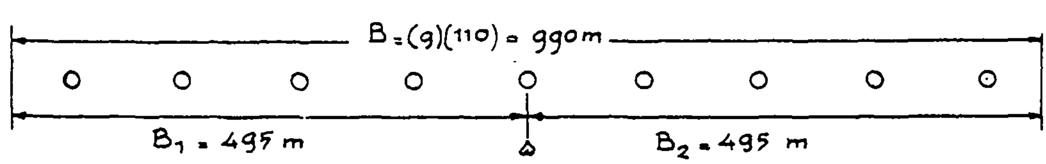
$$h_g^2 - 144 = 108 - 10.8, \quad h_g^2 = 241, \quad h_g = 15.5 \text{ m}$$

and as drawdown at the face of the gallery

$$s_o = h_g - h_o = 15.5 - 11.9 = 3.6 \text{ m}$$

Due to the finite length of the gallery, this drawdown will in reality be smaller

$$s_o' = \beta s_o \quad \text{with} \quad \beta = \frac{1}{2} F_2\left(\frac{B_1}{2L_1}\right) + \frac{1}{2} F_2\left(\frac{B_2}{2L_1}\right)$$



The reduction factor β has its largest value at the place of the centre well

$$B_1 = B_2 = 495 \text{ m}$$

$$\beta = F_2 \left(\frac{495}{600}\right) = F_2 \,(0.825) = 0.676$$

$$s_o' = 2.4 \text{ m} \qquad \text{and} \qquad h_o' = 15.5 - 2.4 = 13.1 \text{ m}$$

In reality the groundwater is not abstracted with a gallery but by means of a line of wells. Due to this point abstraction an additioned drawdown will occur

$$\Delta s = \frac{Q_o}{2\pi k h_o'} \ln \frac{b}{2\pi r_o}$$. In the case under consideration

$$\Delta s = \frac{(4.5)10^{-3}}{2\pi(0.25)10^{-3}\,(13.0)} \ln \frac{110}{2\pi(0.2)} = 0.221 \ln 87.5 = 1.0 \text{ m}$$

giving as lowest water table depth inside the well, neglecting well losses

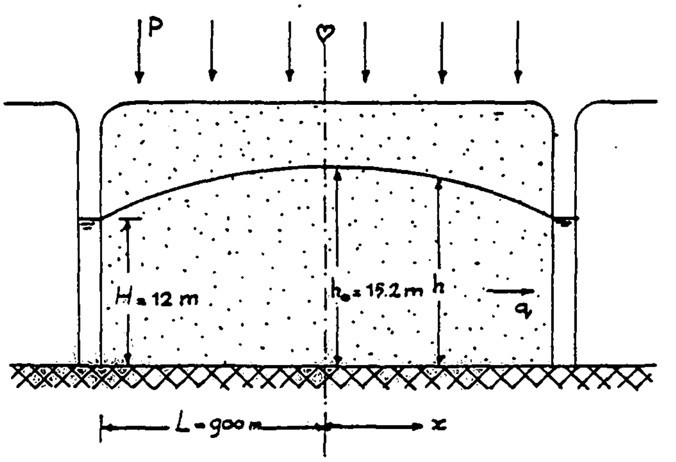
$$h_o'' = h_o' - \Delta s = 13.0 - 1.0 = 12.0 \text{ m}$$

5.13 An unconfined aquifer is situated above an impervious base and is composed of sand with a coefficient of permeability k equal to $(0.15)10^{-3}$ m/sec. Two fully penetrating ditches separate in this aquifer a strip of land of constant width equal to 1800 m. The water level in the ditches is constant at 12 m above the impervious base. Due to recharge by rainfall the water table in the centre of the strip of land is higher by an amount of 3.2 m.

In the strip of land, at a distance of 600 m parallel to one of the ditches, a line of fully penetrating wells is constructed. The wells have outside diameters of 0.30 m, are set at constant intervals b of 100 m and are pumped at a capacity Q_o of $(2.0)10^{-3}$ m^3/sec each.

What is the drawdown at the well face and what is the remaining ground-water outflow to both ditches?

With the notations of the figure at the right, the equations of flow become



Darcy $q = -kh \frac{dh}{dx}$

continuity $q = Px$

combined $h dh = -\frac{P}{k} x dx$

integrated $h^2 = -\frac{P}{k} x^2 + C$

Substitution of the boundary conditions

$$x = 0,\ h = h_o = 15.2 \text{ m};\quad x = L = 900 \text{ m},\quad h = H = 12 \text{ m}$$

gives with $k = (0.15)10^{-3}$ m/sec

$$(15.2)^2 = C \qquad \text{or} \qquad C = 231$$

$$(12)^2 = -\frac{P}{(0.15)10^{-3}}(900)^2 + C \qquad P = (16.1)10^{-9} \text{m/sec}$$

substituted

$$h^2 = -\frac{(16.1)10^{-9}}{(0.15)10^{-3}} x^2 + 231 \qquad \text{or} \quad h^2 = 231 - (\frac{x}{96.5})^2$$

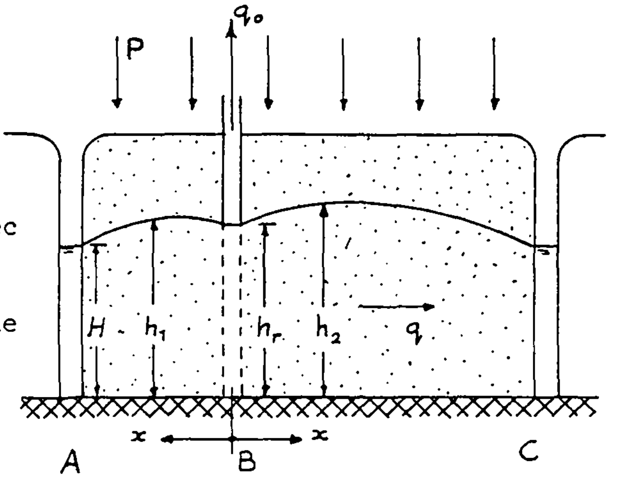
At the proposed line of wells, x = 300 m, giving as water table elevation before pumping

$$h_w^{\ 2} = 231 - (\frac{300}{96.5})^2 = 231 - 10 = 221 \qquad \text{or} \quad h_w = 14.9 \text{ m}$$

When provisionally the line of wells is replaced by a gallery with the same capacity per lineal meter

$$q_o = \frac{Q_o}{b} = \frac{(2.0)10^{-3}}{100}$$

$$q_o = (20)10^{-6} \ m^3/m'/sec$$



the equations of flow for both the strips B - A and B - C read

Darcy $\qquad q = -kh\frac{dh}{dx}$

continuity $\frac{dq}{dx} = P$, $\qquad$ integrated $q = Px + C_1$

combined $\quad hdh = -\frac{P}{k} x\,dx - \frac{C_1}{k} dx$

integrated $\quad h^2 = -\frac{P}{k} x^2 - \frac{2C_1}{k} x + C_2$

Substitution of the boundary conditions yields

strip B - A

$$x = 0,\ h = h_r : h_r^{\ 2} = C_2$$

$$x = 600, h = 12 : 144 = \frac{(16.1)10^{-9}}{(0.15)10^{-3}}(600)^2 - \frac{2C_1}{(0.15)10^{-3}}(600) + C_2$$

from which follows $C_1 = (0.125)10^{-6}\, h_r^2 - (22.8)10^{-6}$ and

$$q_1 = (16.1)10^{-9}x + (0.125)10^{-6}\, h_r^2 - (22.8)10^{-6}$$

strip B - C

$x = 0, \quad h = h_r \quad : h_r^2 = C_2$

$x = 1200,\ h = 12 \qquad 144 = \dfrac{(16.1)10^{-9}}{(0.15)10^{-3}}(1200)^2 - \dfrac{2C_1}{(0.15)10^{-3}}(1200) + C_2$

from which follows $C_1 = (0.063)10^{-6}\, h_r^2 - (18.7)10^{-6}$ and

$$q_2 = (16.1)10^{-9}\, x + (0.063)10^{-6}\, h_r^2 - (18.7)10^{-6}$$

At point B, x = 0 the groundwater flows equal

$$q_{10} = (0.125)10^{-6}\, h_r^2 - (22.8)10^{-6}$$

$$q_{20} = (0.063)10^{-6}\, h_r^2 - (18.7)10^{-6}$$

Together they must equal the abstraction $q_o = (20)10^{-6}$ or

$$-(20)10^{-6} = (0.188)10^{-6}\, h_r^2 - (41.5)10^{-6}$$

$$h_r^2 = 115, \quad h_r = 10.7 \text{ m}$$

The average drawdown in the line of wells thus equals

$$s'_o = h_w - h_r = 14.9 - 10.7 = 4.2 \text{ m}$$

At the face of the well, the drawdown is larger by an amount

$$\Delta s_o = \frac{Q_o}{2\pi kH} \ln \frac{b}{2\pi r_o} = \frac{(2.0)10^{-3}}{2\pi(0.15)10^{-3}(10.7)} \ln \frac{100}{2\pi(0.15)} = 0.20 \ln 106$$

$$\Delta s_o = 0.9 \text{ m}$$

giving as total drawdown at the well face

$$s_o = 4.2 + 0.9 = 5.1 \text{ m}$$

The outflow in the ditches equal the groundwater flows at x = 600 m and x = 1200 m respectively

A: $$q_1 = (16.1)10^{-9}(600) + (0.125)10^{-6}(10.7)^2 - (22.8)10^{-6}$$

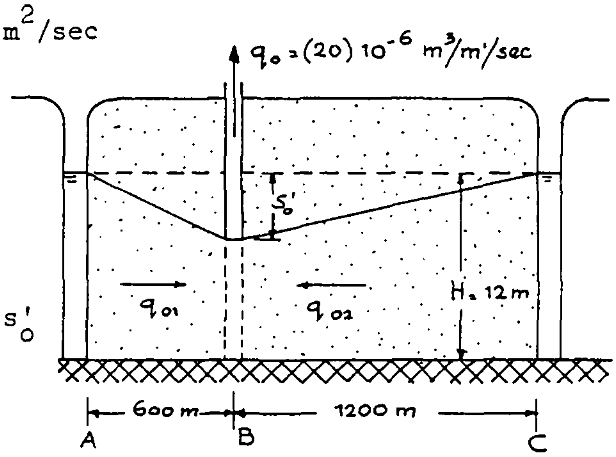
$$q_1 = (9.7)10^{-6} + (14.3)10^{-6} - (22.8)10^{-6} = (1.2)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

C: $$q_2 = (16.1)10^{-9}(1200) + (0.063)10^{-6}(10.7)^2 - (18.7)10^{-6}$$

$$q_2 = (19.3)10^{-6} + (7.2)10^{-6} - (18.7)10^{-6} = (7.8)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

The calculations made above are quite complicated and a computational error will thus easily slip in. A check may be had by neglecting rainfall and assuming the coefficient of transmissibility constant at

$$kH = (0.15)10^{-3}(12) = (1.8)10^{-3} \text{ m}^2/\text{sec}$$



The rate of flows thus become

$$q_{01} = (1.8)10^{-3} \frac{s_o'}{600} = (3)10^{-6} s_o'$$

$$q_{02} = (1.8)10^{-3} \frac{s_o'}{1200} = (1.5)10^{-6} s_o'$$

Together they equal q_o or

$$(20)10^{-6} = (4.5)10^{-6} s_o' , \quad s_o' = 4.4,$$ in close agreement with the value of 4.2 m calculated above. The rates of flow now equal

$$q_{01} = (3)10^{-6}(4.4) = (13.3)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

$$q_{02} = (1.5)10^{-6}(4.4) = (6.7)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

Before pumping, the recharge by rainfall flows out equally to both ditches, giving at each ditch an outflow

$$q = P.L = (16.1)10^{-9}(900) = (14.5)10^{-6}\ m^3/m'/sec$$

Superposition gives as remaining outflows

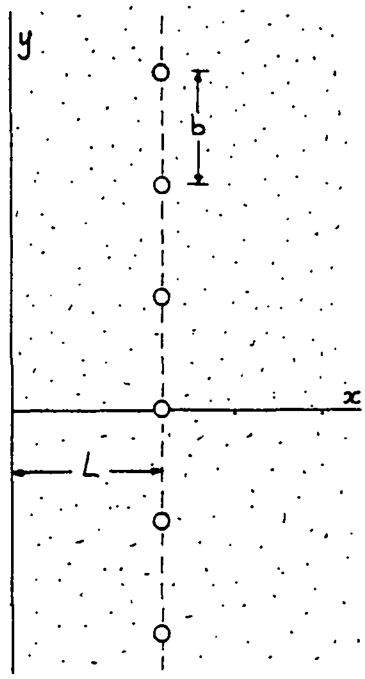
A: $$q = (14.5)10^{-6} - (13.3)10^{-6} = (1.2)10^{-6}\ m^3/m'/sec$$

C: $$q = (14.5)10^{-6} - (6.7)10^{-6} = (7.8)10^{-6}\ m^3/m'/sec$$

in complete agreement with the original calculations.

5.14 An unconfined aquifer is situated above an impervious base and bounded by a fully penetrating ditch. From the aquifer groundwater flows out into this ditch in an amount q_o $m^3/m'/sec$. Parallel to the ditch a line of fully penetrating wells at constant intervals b is constructed. All wells are pumped at equal capacities $Q_o = (0.9) b\, q_o$.

What is the minimum distance between the ditch and the line of wells to prevent water from the ditch to enter the aquifer?



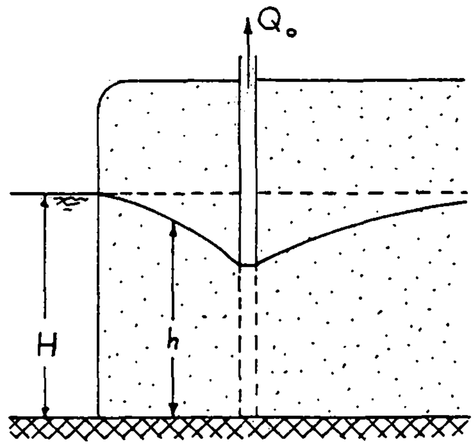
When prior to pumping groundwater is at rest, the water table elevation during pumping is given by the equation

$$H^2 - h^2 = \frac{Q_o}{2\pi k} \ln \frac{\cosh 2\pi(x + L)/b - \cos 2\pi y/b}{\cosh 2\pi(x - L)/b - \cos 2\pi y/b}$$

The slope of the groundwater table perpendicular to the coastline can be found by differentiation to x

$$-2h\,dh = \frac{Q_o}{kb} \left\{ \frac{\sinh 2\pi(x + L)/b}{\cosh 2\pi(x + L)/b - \cos 2\pi y/b} - \right.$$

$$\left. - \frac{\sinh 2\pi(x - L)/b}{\cosh 2\pi(x - L)/b - \cos 2\pi y/b} \right\} dx$$



At the coastline $x = 0$, this formula simplifies to

$$-2H\,dh = \frac{2Q_o}{kb} \frac{\sinh 2\pi L/b}{\cosh 2\pi L/b - \cos 2\pi y/b} dx$$

The slope $-\frac{dh}{dx}$ reaches its maximum value at

$y = 0 + nb$ Substituted

$$-\frac{dh}{dx} = \frac{Q_o}{kHb} \frac{\sinh 2\pi L/b}{\cosh 2\pi L/b - 1}$$

The original outflow of groundwater in a magnitude q_o was accompanied by a slope

$$\frac{dh}{dx} = \frac{q_o}{kH}$$

In the combined case, no water from the ditch will enter the aquifer when the sum of both slopes equal zero or

$$\frac{q_o}{kH} = \frac{Q_o}{kHb} \; \frac{\sinh 2\pi L/b}{\cosh 2\pi L/b - 1}$$

With

$$Q_o = n\, q_o b \quad \text{and}$$

$$\sinh 2\pi L/b = \frac{1}{2}\{e^{2\pi L/b} - e^{-2\pi L/b}\} = \frac{1}{2}\{\alpha - \frac{1}{\alpha}\}$$

$$\cosh 2\pi L/b = \frac{1}{2}\{e^{2\pi L/b} + e^{-2\pi L/b}\} = \frac{1}{2}\{\alpha + \frac{1}{\alpha}\}$$

this requirement may be simplified to

$$1 = n \frac{\alpha - \frac{1}{\alpha}}{\alpha + \frac{1}{\alpha} - 2} \qquad \text{or}$$

$$(1 - n)\alpha^2 - 2\alpha + (1 + n) = 0$$

$$\alpha = \frac{2}{2(1 - n)} \pm \frac{1}{2(1 - n)} \sqrt{4 - 4\,(1 - n)(1 + n)} = \frac{1}{1 - n} \pm \frac{n}{1 - n}$$

and as real solution

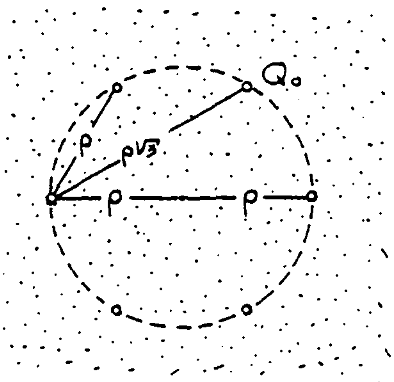
$$\alpha = e^{2\pi L/b} = \frac{1 + n}{1 - n} \qquad \text{For} \quad n = 0.9$$

$$e^{2\pi L/b} = \frac{1.9}{0.1} = 19 = e^{2.95} \qquad L = \frac{2.95b}{2\pi} = 0.47b$$

5.15 An unconfined aquifer of infinite extent has a coefficient of transmissibility kH equal to $(14)10^{-3} m^2/sec$ and is situated above a semi-pervious layer with a resistance c of $(26)10^6$ sec against vertical water movement. Below this semi-pervious layer artesian water with a constant and uniform level is present.

In the unconfined aquifer a circular battery of 6 fully penetrating wells is constructed. The wells are situated on a circle with a diameter of 180 m, have equal intervals, outside diameters of 0.3 m and are pumped at a constant capacity of $(5.5)10^{-3}\ m^3/sec$ each.

What is the lowering of the phreatic watertable in the centre of the battery and at the face of the wells? How much artesian water will percolate upward over the area of the battery as a result of well pumping?



The drawdown due to pumping a single well in an unconfined aquifer above a semi-pervious layer equals

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \quad \text{with } \lambda = \sqrt{kHc}$$

This gives as drawdown in the centre of the battery

$$s_c = \frac{6Q_o}{2\pi kH} K_o\left(\frac{\rho}{\lambda}\right) \qquad \lambda = \sqrt{(14)10^{-3}(26)10^6} = 603 \text{ m}$$

$$s_c = \frac{(6)(5.5)10^{-3}}{2\pi(14)10^{-3}} K_o\left(\frac{90}{603}\right) = 0.375\ K_o\ (0.149) = (0.375)(2.04) = 0.77 \text{ m}$$

and at the well face

$$s_o = \frac{Q_o}{2\pi kH} K_o\left(\frac{r_o}{\lambda}\right) + \frac{2Q_o}{2\pi kH} K_o\left(\frac{\rho}{\lambda}\right) + \frac{2Q_o}{2\pi kH} K_o\left(\frac{\rho\sqrt{3}}{\lambda}\right) + \frac{Q_o}{2\pi kH} K_o\left(\frac{2\rho}{\lambda}\right)$$

With for $\frac{r}{\lambda}$ small

$$K_o\left(\frac{r}{\lambda}\right) = \ln\frac{1.123\lambda}{r}$$

$$s_o = \frac{0.375}{6} \ln \frac{(1.123)(603)}{0.15} + \frac{0.375}{3} K_o \left(\frac{90}{603}\right) + \frac{0.375}{3} K_o \left(\frac{90\sqrt{3}}{603}\right) + \frac{0.375}{6} K_o\left(\frac{180}{603}\right)$$

$$s_o = \frac{0.375}{6} \{\ln 4502 + K_o (0.299)\} + \frac{0.375}{3} \{K_o(0.149) + K_o(0.259)\}$$

$$s_o = \frac{0.375}{6} (8.41 + 1.38) + \frac{0.375}{3} (2.04 + 1.51) = 0.61 + 0.44 = 1.05 \text{ m}$$

The average drawdown s' in the circular line of wells is the drawdown s_o at the well face minus the influence Δs_o of point abstraction

$$\Delta s_o = \frac{Q_o}{2\pi kH} \ln \frac{b}{2\pi r_o} \quad \text{with b as interval between the wells}$$

$$s_o = \frac{0.375}{6} \ln \frac{90}{2\pi(0.15)} = \frac{0.375}{6} \ln 95.6 = 0.28 \text{ m}$$

$$s' = s_o - \Delta s_o = 1.05 - 0.28 = 0.77 \text{ m}$$

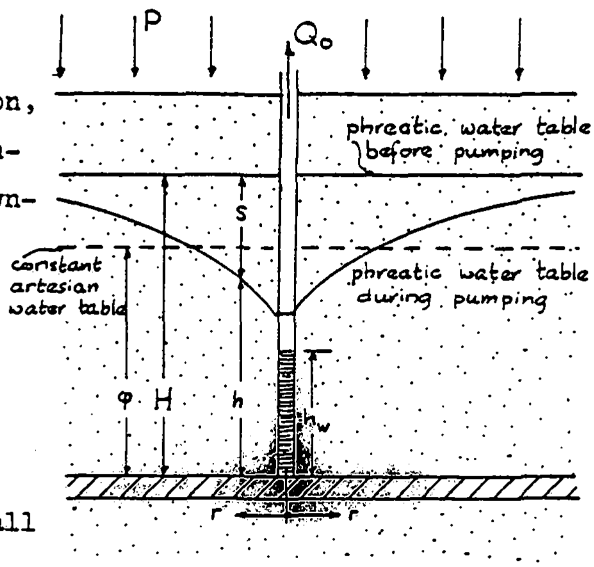
This drawdown is the same as the drawdown s_c in the centre of the battery, meaning that over the full area of the battery the drawdown will have this value. The upward percolation of artesian water thus equals

$$Q = \pi\rho^2 \frac{s'}{c} = \pi(90)^2 \frac{0.77}{(26)10^6} = (0.75)10^{-3} \text{ m}^3/\text{sec}$$

5.16 A horizontal semi-pervious layer separates a confined and an unconfined aquifer, both of infinite extent. The piezometric level of the aquifers is constant, reaching in the unconfined aquifer to 15 m and in the confined aquifer to 13.4 m above the top of the semi-pervious layer. The unconfined aquifer has a coefficient of permeability k equal to $(0.45)10^{-3}$ m/sec and is recharged by rainfall P in an amount of $(18)10^{-9}$ m/sec. The confined aquifer has a very great depth and consists of coarse sand/fine gravel.

In the unconfined aquifer 5 fully penetrating wells are constructed in a straight line, at constant intervals of 60 m. The well screens have diameters of 0.4 m and extend from the top of the semi-pervious layer 7 m upward. The wells are pumped at constant rates Q_o of $(8)10^{-3}$ m^3/sec each.

What is the maximum drawdown at the well face?

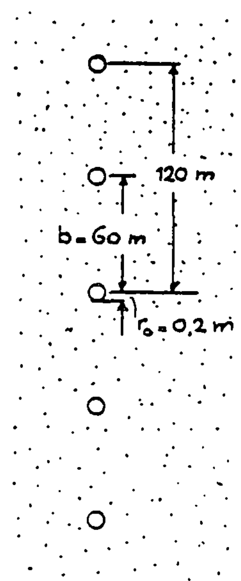


In the case under consideration, the artesian water table may be considered constant, giving as drawdown-distance relationship for a fully penetrating well in the unconfined aquifer

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \quad \text{with}$$

$$\lambda = \sqrt{kHc} \quad \text{and for } \frac{r}{\lambda} \text{ small}$$

$$K_o\left(\frac{r}{\lambda}\right) = \ln \frac{1.123\lambda}{r}$$



The maximum drawdown occurs at the face of the centre well. Using the method of superposition, this drawdown equals

$$s_o = \frac{Q_o}{2\pi kH} \left\{\ln \frac{1.123\lambda}{r_o} + 2K_o\left(\frac{b}{\lambda}\right) + 2K_o\left(\frac{2b}{\lambda}\right)\right\}$$

To determine the resistance c of the semi-pervious layer against vertical water movement, it is considered that with a horizontal water table in the unconfined aquifer, the full recharge by rainfall must penetrate downward to the artesian aquifer (of great transmissivity) below. The accompanying loss of head equals the difference in piezometric level, thus

$$\Delta = Pc = H - \phi \quad \text{or}$$

$$(18)10^{-9}c = 15 - 13.4 \qquad c = \frac{1.6}{(18)10^{-9}} = (89)10^{6} \text{ sec.}$$ This gives

$$\lambda = \sqrt{(0.45)10^{-3}\,(15)(89)10^{6}} = 775 \text{ m}$$

$$s_o = \frac{(8)10^{-3}}{2\pi(0.45)10^{-3}\,(15)} \{\ln \frac{(1.123)(775)}{0.2} + 2K_o\,(\frac{60}{775}) + 2K_o(\frac{120}{775})\}$$

$$s_o = 0.189\,\{\ln 4350 + 2K_o\,(0.0774) + 2K_o\,(0.155)\}$$

$$s_o = 0.189\,\{8.38 + 2(2.68) + 2(2.00)\} = (0.189)(17.74) = 3.36 \text{ m}$$

In reality, however, the well only partially penetrates the aquifer, giving rise to an additional drawdown

$$\Delta s_o = \frac{Q_o}{2\pi kH}\,\frac{1-p}{p}\,\ln\,\frac{(1-p)h_w}{2r_o}, \quad \text{with } p = \frac{h_w}{h_o} = \frac{h_w}{H - s_o}$$

$$p = \frac{7}{15 - 3.36} = \frac{7}{11.64} = 0.60$$

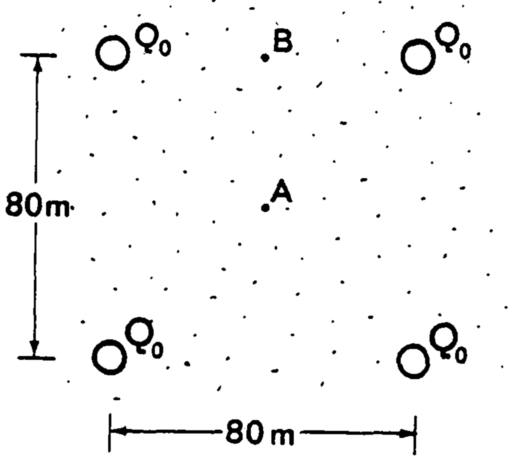
$$\Delta s_o = (0.189)\,\frac{0.40}{0.60}\,\ln\,\frac{(0.40)(7)}{0.4} = 0.25 \text{ m}$$

The maximum drawdown at the face of the partially penetrating well thus becomes

$$s_o' = s_o + \Delta s_o = 3.36 + 0.25 = 3.6 \text{ m}$$

5.17 An unconfined aquifer of infinite extent has a coefficient of transmissibility kH equal to 0.01 m^2/sec and is situated above a semi-pervious layer with a resistance c of $(36)10^6$ sec against vertical water movement. Below this less pervious layer artesian water is present at a constant and uniform level. For the construction of a building pit with a size of 80 x 80 m, a lowering of the phreatic water table is necessary. This will be accomplished by setting 4 wells, one at each corner of the pit and pumping these wells at equal capacities.

What is the minimum amount of groundwater abstraction necessary to assure that over the full area of the pit the lowering of the phreatic water table is at least 3 m.



The drawdown accompanying the flow of groundwater to a well in an unconfined aquifer above a semi-pervious layer is given by

$$s = \frac{Q_o}{2\pi kH} K_o\left(\frac{r}{\lambda}\right) \text{ with}$$

$$\lambda = \sqrt{kHc} = \sqrt{(0.01)(36)10^6} = 600 \text{ m}$$

For $\frac{r}{\lambda} < 0.16$ or $r < 100$ m the drawdown formula may be replaced by

$$s = \frac{Q_o}{2\pi kH} \ln \frac{1.123\ \lambda}{r} = \frac{Q_o}{2\pi kH} \ln \frac{674}{r}$$

This gives as drawdown in points A and B

$$s_A = \frac{Q_o}{2\pi kH} \ln \frac{(674)^4}{(40\sqrt{2})^4} = 9.91 \frac{Q_o}{2\pi kH}$$

$$s_B = \frac{Q_o}{2\pi kH} \ln \frac{(674)^4}{(40)^2(40\sqrt{5})^2} = 9.69 \frac{Q_o}{2\pi kH}$$

The drawdown is lowest in point B, giving as requirement

$$Q_o > \frac{2\pi kHs}{9.69},\ Q_o > \frac{2\pi(0.01)(3)}{9.69},\ Q_o > (19.45)10^{-3} \text{ m}^3/\text{sec} \quad \text{and}$$

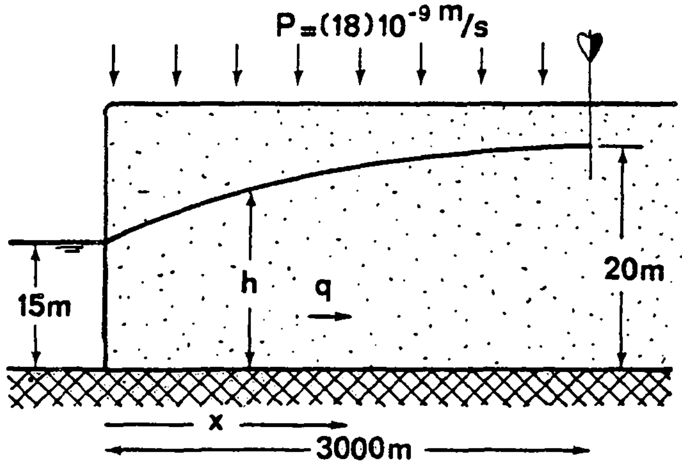
$$4Q_o = (77.8)10^{-3} \text{ m}^3/\text{sec} = (280) \text{ m}^3/\text{hour}$$

5.18 A semi-infinite unconfined aquifer is situated above a horizontal impervious base and is bounded by a fully penetrating ditch, while at a distance of 3000 m parallel to the ditch a water divide is present. The water level in the ditch is constant at 15 m above the base, while due to recharge by rainfall P in an amount of $(18)10^{-9}$ m/sec the water table at the water divide rises to 20 m above the base.

At a distance of 500 m parallel to the ditch a line of fully penetrating wells is constructed, consisting of 9 units with outer diameters of 0.4 m, at intervals of 200 m. The wells are pumped at constant rates Q_o of $(15)10^{-3}$ m^3/sec each.

Calculate the lowest water level in the line of wells when it may be assumed that the position of the water divide relative to the ditch remains unchanged.

To calculate the unknown value of the coefficient of permeability k, the flow pattern shown at the right must first be analysed



Darcy $q = -kh \frac{dh}{dx}$

continuity $\frac{dq}{dx} = P$ or $q = Px + C_1$

combined $hdh = -\frac{P}{k} xdx - \frac{C_1}{k} dx$

integrated $h^2 = -\frac{P}{k} x^2 - \frac{2C_1}{k} x + C_2$

Substitution of the boundary condition gives

$x = 0$, $h = 15$ m, $225 = C_2$

$x = 3000$ m, $h = 20$ m, $400 = -\frac{0.162}{k} - 6000 \frac{C_1}{k} + C_2$

$x = 3000$ m, $q = 0$ $0 = (54)10^{-6} + C_1$

or $C_1 = -(54)10^{-6}$, $C_2 = 225$ and

$$400 = -\frac{0.162}{k} + \frac{0.324}{k} + 225, \quad \frac{0.162}{k} = 175$$

$$k = \frac{0.162}{175} = (0.926)10^{-3} \text{ m/sec}$$

At the site of the future line of wells (x = 500 m), the water table elevation follows from

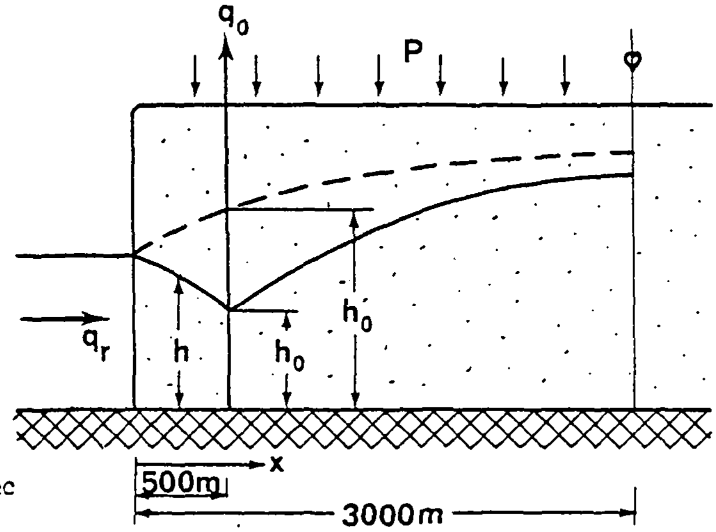
$$(h_o')^2 = -\frac{(18)10^{-9}}{(0.926)10^{-3}}(500)^2 + \frac{(2)(54)10^{-6}}{(0.926)10^{-3}}(500) + 225$$

$$(h_o')^2 = -4.86 + 58.32 + 225 = 278.46 \quad , \quad h_o' = 16.69 \text{ m}$$

To calculate the drawdown due to pumping the line of wells, this line is first replaced by a fully penetrating gallery of infinite length with as capacity

$$q_o = \frac{Q}{b} = \frac{(15)10^{-3}}{200}$$

$$= (75)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$



The total recharge by rainfall amounts to

$$P.L = (18)10^{-9}(3000) = (54)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

requiring an inflow of water from the bounding ditch with as magnitude

$$q_r = (75)10^{-6} - (54)10^{-6} = (21)10^{-6} \text{ m}^3/\text{m}'/\text{sec}$$

The equations of flow remains the same as derived above

$$q = Px + C_1$$
$$h^2 = -\frac{P}{k}x^2 - \frac{2C_1}{k}x + C_2$$

but the boundary conditions are different

$x = 0$, $q = q_r = (21)10^{-6} = C_1$ or $C_1 = (21)10^{-6}$

$x = 0$, $h = 15$ m $225 = C_2$ or $C_2 = 225$

This gives for x = 500 m

$$h_o^2 = -\frac{(18)10^{-9}}{(0.926)10^{-3}}(500)^2 - \frac{(2)(21)10^{-6}}{(0.926)10^{-3}}(500) + 225$$

$$h_o^2 = -4.86 - 22.68 + 225 = 197.46 \quad , \quad h_o = 14.05 \text{ m}$$

and as drawdown

$$s' = h' - h = 16.69 - 14.05 = 2.64 \text{ m}$$

The line of wells has only a limited length, reducing the drawdown to

$$s_o = \beta s'_o \text{ with for the centre well}$$

$$\beta = F_2\left\{\frac{(4.5)(200)}{(2)(500)}\right\} = F_2(0.900) = 0.697$$

$$s_o = (0.697)(2.64) = 1.84 \text{ m}$$ and augmenting the water table depth to

$$h_{oo} = 14.05 + (2.64 - 1.84) = 14.85 \text{ m}$$

Due to point abstraction, the drawdown at the face of the well is larger by

$$\Delta s_o = \frac{Q_o}{2\pi kH} \ln \frac{b}{2\pi r_o} = \frac{(15)10^{-3}}{2\pi(0.926)10^{-3}(14.85)} \ln \frac{200}{2\pi(0.2)} = 0.88 \text{ m}$$

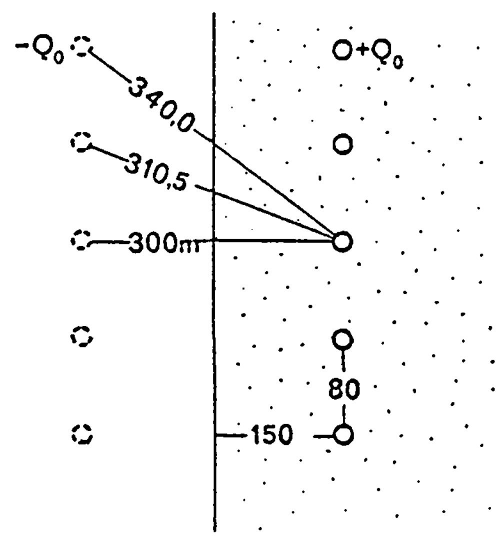
giving as final table elevation at the well face

$$h_{ooo} = 14.85 - 0.88 = 13.97 \text{ m}$$

5.19 An unconfined aquifer has a coefficient of permeability k equal to $(0.3)10^{-3}$ m/sec and is situated above an impervious base. To the left the aquifer is bounded by a fully penetrating ditch, to the right it extends to infinity. The water level in the ditch is constant at 25 m above the impervious base.

At a distance of 150 m from the ditch, a line of 5 fully penetrating wells at intervals of 80 m and outside diameters of 0.4 m is constructed. From each well groundwater in an amount $Q_o = (12)10^{-3}$ m^3/sec is abstracted.

What is the maximum lowering of the ground-water table at the well face? At what distance from the shoreline is the lowering of the groundwater table less than 0.05 m?



The maximum lowering of the groundwater table will occur at the face of the centre well. Using the method of images, the remaining water table depth h_o is given by

$$H^2 - h_o^2 = \frac{Q_o}{\pi k} \Sigma \ \ln \frac{r'}{r}$$

$$(25)^2 - h_o^2 = \frac{(12)10^{-3}}{\pi(0.3)10^{-3}} \left(\ln \frac{300}{0.2} + 2 \ln \frac{310.5}{80} + 2 \ln \frac{340.0}{160}\right)$$

$$625 - h_o^2 = 146.84 \ , \quad h_o^2 = 478.16 \ , \quad h_o = 21.87 \quad \text{and}$$

$$s = H - h_o = 25 - 21.87 = 3.13 \text{ m}$$

When the drawdown of 0.05 m occurs at a distance x from the shoreline, the distance between this point and the real wells equals x - 150 m and the distance to the image wells x + 150 m. This gives

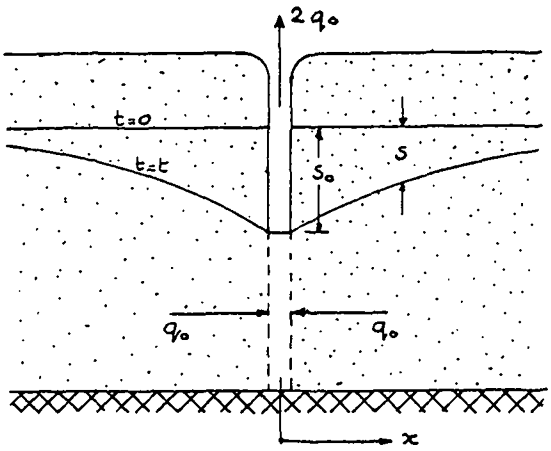
$$(25)^2 - (24.95)^2 = 5 \frac{(12)10^{-3}}{\pi(0.3)10^{-3}} \ln \frac{x + 150}{x - 150} \ , \quad \frac{x + 150}{x - 150} = 1.040 \quad \text{and}$$

$$x = 150 \frac{1.040 + 1}{1.040 - 1} = 7650 \text{ m}$$

This distance is so large, that indeed the abstraction by the line of wells may be concentrated in the centre.

5.21 An unconfined aquifer of infinite extent has a coefficient of transmissibility kH equal to $(4)10^{-3}$ m^2/sec, a specific yield μ of 25% and is situated above an impervious base. In this aquifer an infinite line of fully penetrating wells is constructed. The wells have external diameters of 0.3 m, are set at intervals b of 100 m and are pumped at a constant rate Q_o of $(6)10^{-3}$ m^3/sec.

What is the drawdown at the well face after 100 days of pumping and what is the drawdown at 200 m from the line of wells at this moment?



When provisionally the line of wells is replaced by a gallery with the same capacity per lineal meter

$$2q_o = \frac{Q_o}{b}$$

the drawdown equals

$$s_o = \frac{2q_o}{\sqrt{\pi}} \frac{1}{\sqrt{\mu kH}} \sqrt{t} = \frac{Q_o}{\sqrt{\pi} b} \frac{\sqrt{t}}{\sqrt{\mu kH}}$$

$s = s_o E_3$ with E_3 a function of

$$u = \frac{1}{2} \sqrt{\frac{\mu}{kH}} \frac{x}{\sqrt{t}}$$

After t = 100 days = $(8.64)10^6$ sec, the drawdowns become

$$s_o = \frac{(6)10^{-3}}{\sqrt{\pi}(100)} \frac{\sqrt{(8.64)10^6}}{\sqrt{(0.25)(4)10^{-3}}} = 3.15 \text{ m}$$

At x = 200 m

$$u = \frac{1}{2}\sqrt{\frac{0.25}{(4)10^{-3}}} \frac{200}{\sqrt{(8.64)10^6}} = 0.270, \quad E_3(0.270) = 0.594$$

$$s = (3.15)(0.594) = 1.9 \text{ m}$$

Replacing the gallery by a line of wells gives an additional drawdown at the well face

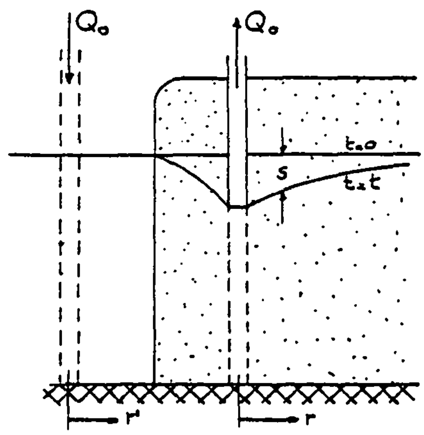
$$\Delta s_o = \frac{Q_o}{2\pi kH} \ln \frac{b}{2\pi r_o} = \frac{(6)10^{-3}}{2\pi(4)10^{-3}} \ln \frac{100}{2\pi(0.15)} = 0.239 \ln 106 \quad \text{or}$$

$\Delta s_o = 1.11$ m Together

$$s'_o = s_o + \Delta s_o = 3.15 + 1.11 = 4.3 \text{ m}$$

5.22 A semi-infinite unconfined aquifer is situated above an impervious base and bounded by a fully penetrating ditch. The coefficient of transmissibility kH below water table amounts to $(8)10^{-3}$ m^2/sec the specific yield μ to 30%. In the aquifer 3 fully penetrating wells are set at intervals of 60 m in a line at a distance of 200 m parallel to the ditch. The wells have outside diameters of 0.6 m and are pumped for a period of 50 days at a constant rate of $(12)10^{-3}$ m^3/sec each.

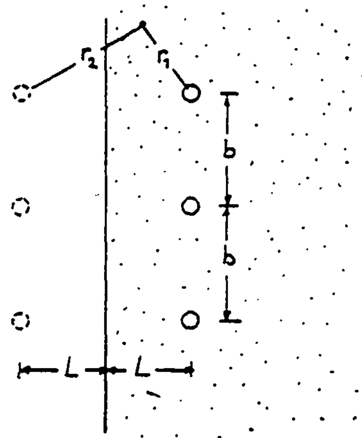
What is the maximum drawdown at the well face at the end of the pumping period?



The unsteady drawdown due to pumping a single well in a semi-infinite unconfined aquifer above an impervious base equals

$$s = \frac{Q_o}{4\pi kH}\{W(u_1^{\,2}) - W(u_2^{\,2})\} \quad \text{with}$$

$$u_1^{\,2} = \frac{\mu}{4kH}\frac{r_1^{\,2}}{t} \qquad u_2^{\,2} = \frac{\mu}{4kH}\frac{r_2^{\,2}}{t}$$



After 50 days $= (4.32)10^6$ sec

$$u^2 = \frac{0.3}{(4)(8)10^{-3}}\,\frac{r^2}{(4.32)10^6} = \frac{r^2}{(0.46)10^6}$$

The maximum drawdown occurs at the face of the centre well. Using the method of superposition this drawdown becomes

$$s_o = \frac{(12)10^{-3}}{4\pi(8)10^{-3}}\left\{W\left(\frac{(0.3)^2}{(0.46)10^6}\right) + 2W\left(\frac{(60)^2}{(0.46)10^6}\right) - W\left(\frac{(400)^2}{(0.46)10^6}\right) - \right.$$

$$\left. - 2W\left(\frac{(60)^2+(400)^2}{(0.46)10^6}\right)\right\}$$

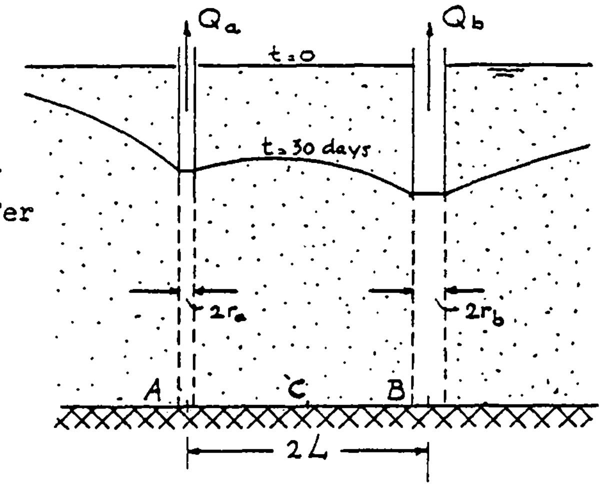
$$s_o = 0.119\,\{W\left[(1.95)10^{-7}\right] + 2W\left[(7.82)10^{-3}\right] - W\left[(3.48)10^{-1}\right] -$$

$$- 2W\left[(3.56)10^{-1}\right]\}$$

$$s_o = 0.119\,\{14.86 + 2(4.28) - 0.80 - 2(0.78)\} = (0.119)(21.06) = 2.5$$

5.23 An unconfined aquifer of infinite extent is situated above an impervious base. The aquifer consists of fractured limestone with a coefficient of transmissibility kH of $(8)10^{-3}$ m^2/sec below water table and a specific yield μ of 5%. Due to absence of recharge the groundwater table is horizontal. In the unconfined aquifer two fully penetrating wells are constructed at an interval of 1000 m. One well has a diameter of 0.3 m and a capacity of $(25)10^{-3}$ m^3/sec, the other well a diameter of 0.6 m and a capacity of $(50)10^{-3}$ m^3/sec. Starting at t = 0, both wells are pumped during a period of 30 days.

What is the drawdown at the face of each well and halfway between the two wells at the end of the pumping period? How much time must elapse before the drawdown halfway the two wells has decreased to 0.1 m ?



The drawdown due to pumping a single well in an unconfined aquifer of infinite extent is given by

$$s = \frac{Q_o}{4\pi kH} W(u^2) \qquad \text{with}$$

$$u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

The drawdown caused by pumping the wells A and B can be found with the method of superposition

$$s = \frac{Q_a}{4\pi kH} W(u_a{}^2) + \frac{Q_b}{4\pi kH} W(u_b{}^2).$$ This gives at

$$t = 30 \text{ days} = (2.59)10^6 \text{ sec}$$

well face A:

$$u_A{}^2 = \frac{0.05}{(4)(8)10^{-3}} \frac{(0.15)^2}{(2.59)10^6} = (1.36)10^{-8} \qquad W(u_a)^2 = 17.54$$

$$u_B{}^2 = \frac{0.05}{(4)(8)10^{-3}} \frac{(1000)^2}{(2.59)10^6} = 0.60 \qquad W(u_b{}^2) = 0.454$$

$$s_A = \frac{(25)10^{-3}}{4\pi(8)10^{-3}}(17.54) + \frac{(50)10^{-3}}{4\pi(8)10^{-3}}(0.454)$$

$$s_A = (0.249)(17.54) + (0.498)(0.454) = 4.37 + 0.23 = 4.6 \text{ m}$$

well face B:

$$u_A^2 = \frac{0.05}{(4)(8)10^{-3}} \frac{(1000)^2}{(2.59)10^6} = 0.60 \qquad W(u_a^2) = 0.454$$

$$u_B^2 = \frac{0.05}{(4)(8)10^{-3}} \frac{(0.3)^2}{(2.59)10^6} = (5.43)10^{-8} \qquad W(u_b^2) = 16.15$$

$$s_B = (0.249)(0.454) + (0.498)(16.15) = 0.11 + 8.05 = 8.2 \text{ m}$$

halfway

$$u^2 = \frac{0.05}{(4)(8)10^{-3}} \frac{(500)^2}{(2.59)10^6} = 0.15 \qquad W(u^2) = 1.465$$

$$s_C = (0.249)(1.465) + (0.498)(1.465) = (0.747)(1.465) = 1.1 \text{ m}$$

Stopping the abstraction at the end of the pumping period must mathematically be obtained by superimposing a recharge of the same magnitude. When the time t is measured from the moment pumping starts and the duration of the pumping period is called τ (equal to 30 days or $(2.59)10^6$ sec) the drawdown in point C now equals

$$s_C = (0.747)\ W\ (u_t^2) - (0.747)\ W\ (u_{t-\tau}^2)$$

Before the drawdown has receded to 0.1 m, much time will have elapsed. The values of u^2 are than so small that the formula above may be approximated by

$$s_C = (0.747) \ln \frac{0.562}{u_t^2} - (0.747) \ln \frac{0.562}{u_{t-\tau}^2} = 0.747 \ln \frac{u_{t-\tau}^2}{u_t^2}$$

With $u^2 = \frac{\mu}{4kH} \frac{r^2}{t}$ this formula simplifies to

$$s_C = 0.747 \ln \frac{t}{t-\tau}$$

A value $s_C = 0.1$ m consequently requires

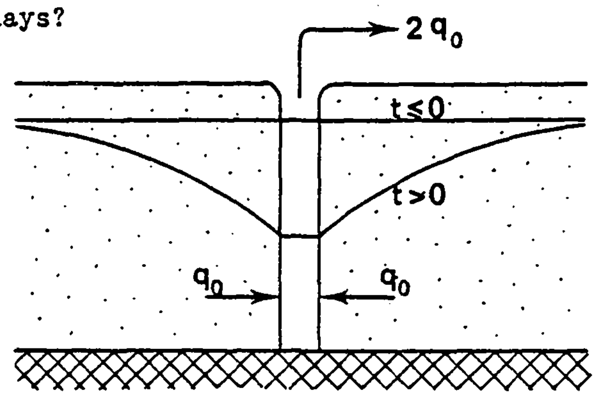
$$\ln \frac{t}{t-\tau} = 0.134, \ \frac{t}{t-\tau} = 1.143, \quad t = 8\tau, \quad t-\tau = 7\tau$$

After pumping has stopped, (7)(30) or 210 days must elapse before the drawdown halfway both wells has decreased to 0.1 m.

5.24 An unconfined aquifer has a coefficient of transmissibility kH equal to 0.02 m^2/sec and a specific yield μ of 30% and is situated above an impervious base. In this aquifer a ditch is constructed from which water is abstracted according to the following pattern

$t < 0$	$q = 0$
$0 < t < 10$ days	$q = (0.8)10^{-3}$ m^3/m'/sec
$10 < t < 20$ days	$q = (1.2)10^{-3}$ m^3/m'/sec
$20 < t$	$q = 0$

What is the lowering of the groundwater table at a distance of 60 m from the ditch at t = 50 days?



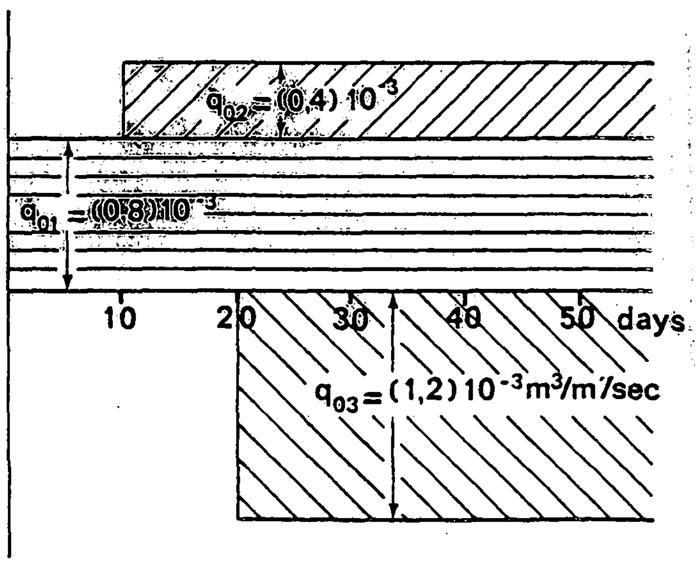
For the situation sketched in the figure at the right, the drawdown is given by

$x = 0 \quad s_o = \frac{2q_o}{\sqrt{\pi}}\frac{1}{\sqrt{\mu kH}}\sqrt{t}$

$x = x \quad s = s_oE_3$ with E_3 a function of the parameter

$u = \frac{1}{2}\sqrt{\frac{\mu}{kH}}\frac{x}{\sqrt{t}}$

These formula hold true for a constant abstraction, starting at t = 0 and thereafter continuing indefinitely. To apply these formula for the case under consideration, superposition is necessary as indicated in the diagram on the right.



With the data supplied

$$s_o = (7.28)(2q_o)\sqrt{t}$$

$$u = \frac{116.2}{\sqrt{t}} \quad \text{and}$$

$t = 50$ days $= (4.320)10^6$ sec

$t = (50 - 10)$ days $= (3.456)10^6$ sec

$t = (50 - 20)$ days $= (2.592)10^6$ sec

the drawdown at x = 60 m, t = 50 days becomes

$$s = (7.28)(0.8)10^{-3}\sqrt{(4.320)10^6}\ E_3\left(\frac{116.2}{\sqrt{(4.320)10^6}}\right) +$$

$$+ (7.28)(0.4)10^{-3}\sqrt{(3.456)10^6}\ E_3\left(\frac{116.2}{\sqrt{(3.456)10^6}}\right) -$$

$$- (7.28)(1.2)10^{-3}\sqrt{(2.592)10^6}\ E_3\left(\frac{116.2}{\sqrt{(2.592)10^6}}\right)$$

$$s = 12.10\ E_3(0.0559) + (5.41)E_3(0.0625) - (14.06)E_3(0.722)$$

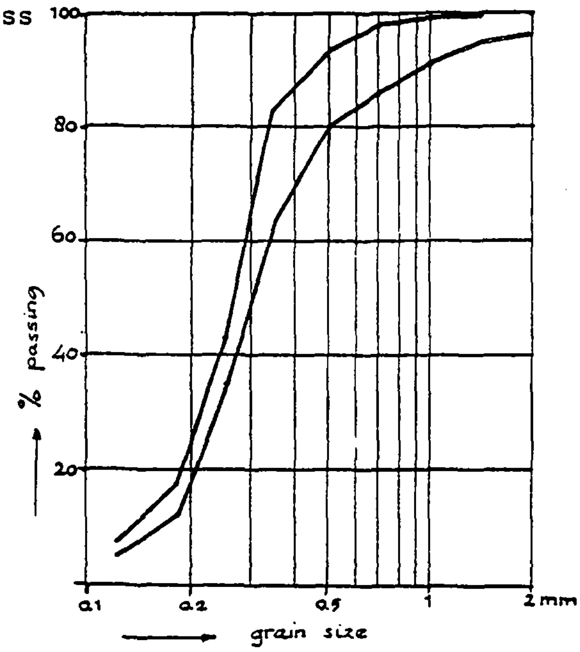
Taking the values of E_3 from Groundwater Recovery, page 42, gives

$$s = (12.10)(0.9041) + (5.41)(0.8931) - (14.06)(0.8773)$$

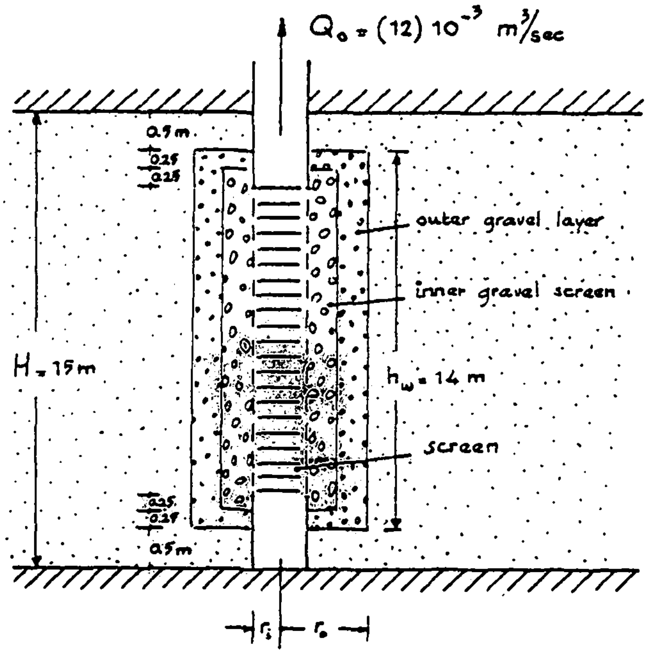
$$s = 10.94 + 4.83 - 12.33 = 3.44\ \text{m}$$

6.01 An artesian aquifer has a thickness of 15 m and is composed of sand, the extreme grain size distributions of which are shown in the diagram at the right. In this aquifer a well must be constructed for a capacity Q_o of $(12)10^{-3}$ m^3/sec.

Sketch the well construction to be applied and calculate all necessary dimensions.



With regard to the small aquifer depth and the fine grainsize distribution of the aquifer material, a fully penetrating artificially gravel-packed well must be applied, as sketched in the picture at the right. With 40% of the finest aquifer material smaller than $(0.24)10^{-3}$ m, the maximum allowable entrance velocity according to Gross equals



$$v_a = (2)(0.24)10^{-3} \text{ or}$$

$$v_a = (0.48)10^{-3} \text{ m/sec}$$

The outer diameter of the gravel pack now follows from

$$Q_o = 2\pi r_o h_w v_a$$

$$r_o = \frac{Q_o}{2\pi h_w v_a} = \frac{(12)10^{-3}}{2\pi(14)(0.48)10^{-3}} = 0.284 \text{ m, rounded of}$$

$$r_o = 0.3 \text{ m}, \; 2r_o = 0.6 \text{ m}$$

With a double gravel treatment, each layer 0.07 m thick, the inner diameter becomes

$2r_i = 0.6 - (4)(0.07) = 0.32$ m, giving as maximum velocity of upward flow inside the screen

$$v_{max} = \frac{Q_o}{\pi r_i^2} = \frac{(12)10^{-3}}{\pi(0.16)^2} = 0.15 \text{ m}$$

This velocity is so small that the inner diameter may be reduced to 0.25 m, increasing the thickness of the gravel layers to nearly 9 cm each and augmenting the maximum velocity of upward water movement to only

$$v_{max} = \frac{(12)10^{-3}}{\pi(0.125)^2} = 0.24 \text{ m/sec.}$$

As regards the composition of the gravel pack, the lower limit of the outer layer should have a size not exceeding 4 times the 85% diameter of the finest aquifer material or

$$(4)(0.38) = 1.52 \text{ mm}$$

and the upper layer not more than a factor $\sqrt{2} = 1.41$ coarser or

$$(1.41)(1.52) = 2.14 \text{ mm}$$

As commercially available material, the size 1.4 - 2 mm will be chosen. The inner layer is again a factor 4 coarser or 5.6 - 8 mm allowing slot openings of 2 mm.

7.01 In a confined aquifer of large extent, a test well is pumped at a constant rate Q_o of $(8)10^{-3}$ m^3/sec. The steady - state drawdown is measured with the help of piezometers at 20 and 60 m from the testwell and amounts to 1.05 and 0.72 m respectively.

What is the coefficient of transmissibility of the aquifer concerned?

According to Thiem's formula, the difference in drawdown between two points at distances of r_1 and r_2 from the well centre equals

$$s_1 - s_2 = \frac{Q_o}{2\pi kH} \ln \frac{r_2}{r_1}$$

From this formula follows as coefficient of transmissibility

$$kH = \frac{Q_o}{2\pi(s_1 - s_2)} \ln \frac{r_2}{r_1}$$

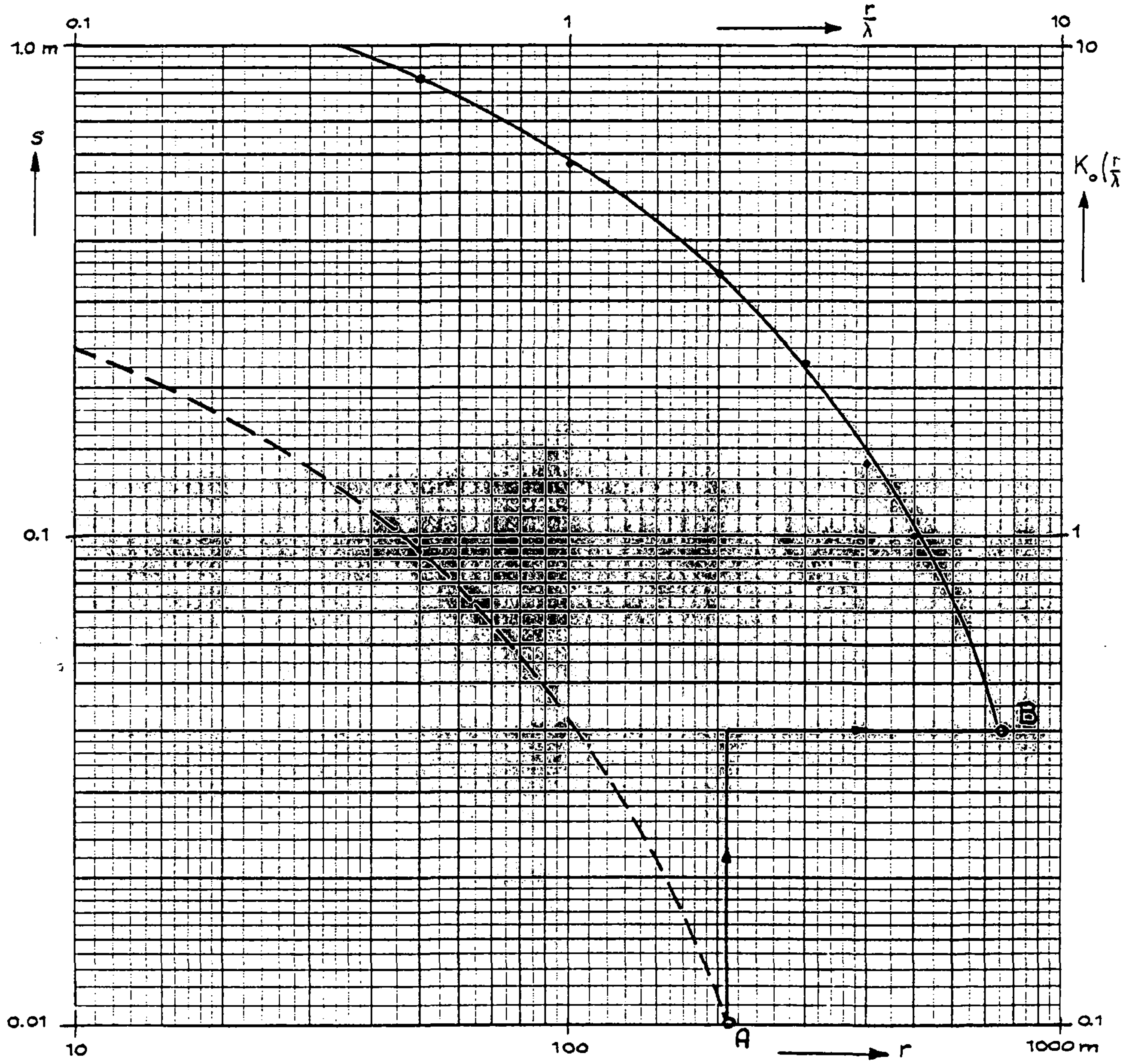
Substitution of the data gives

$$kH = \frac{(8)10^{-3}}{2\pi(1.05 - 0.72)} \ln \frac{60}{20} = \frac{(8)10^{-3}\ (1.10)}{2\pi(0.33)} = (4.2)10^{-3}\ m^2/sec$$

7.02 A leaky artesian aquifer is situated above an impervious base and topped by a semi-pervious layer, above which phreatic water at a constant level is present. In the aquifer a testwell is constructed and pumped at a constant rate Q_o of $(17)10^{-3}$ m^3/sec. The resulting steady -state drawdown s of the artesian water table is measured at various distances r from the centre of the pumped well:

r = 50	100	200	300	400	500	700	m
s = 0.85	0.58	0.34	0.22	0.14	0.10	0.05	m

What are the values of the geo-hydrological constants for this formation?



With a leaky artesian aquifer the drawdown s as function of the distance r is given by

$$s = \frac{Q_o}{2\pi kH} K_o(\frac{r}{\lambda})$$

In the accompanying diagram with logarithmic divisions on both axis, the drawdown s is plotted against the distance r, while the Bessel-function $K_o(\frac{r}{\lambda})$ is indicated with a dotted line. To cover the plotted points as well as possible, the dotted line must be moved upwards and sideways, so that point A arrives in point B. The coordinates of both points

A: $\frac{r}{\lambda} = 2.1$ $\quad K_o(\frac{r}{\lambda}) = 0.1$

B: $r = 760$ m $\quad s = 0.04$ m

must now correspond, giving as relations

$$\frac{760}{\lambda} = 2.1 \quad \text{or} \quad \lambda = 360 \text{ m}$$

$$0.04 = \frac{(17)10^{-3}}{2\pi kH}(0.1) \quad \text{or} \quad kH = (6.8)10^{-3} \text{ m}^2/\text{sec}$$

and

$$c = \frac{\lambda^2}{kH} = \frac{(360)^2}{(6.8)10^{-3}} = (19)10^6 \text{ sec}$$

7.11 A semi-infinite unconfined aquifer has a saturated thickness of 15 m, is situated above an impervious base and bounded by a ditch. In a line perpendicular to the ditch a well and two piezometers are constructed. The well is situated at 400 m from the ditch, while the piezometers are at distances r of 20 m and 50 m more inland. The well is pumped at a constant rate Q_o of $(30)10^{-3}$ m^3/sec. After reaching steady-state conditions the drawdown s in the piezometers amounts to 2.20 m and 1.67 m respectively.

What is the coefficient of permeability of the unconfined aquifer?

For an unconfined aquifer above an impervious base, the general well formula reads

$$H^2-h^2 = \frac{Q_o}{\pi k} \ln \frac{R}{r}$$

Assuming that R has the same value for both piezometers gives

$$h_2^2-h_1^2 = \frac{Q_o}{\pi k} \ln \frac{r_2}{r_1} \qquad \text{With}$$

$$h_1 = 15 - 2.20 = 12.80 \quad \text{and} \quad h_2 = 15 - 1.67 = 13.33$$

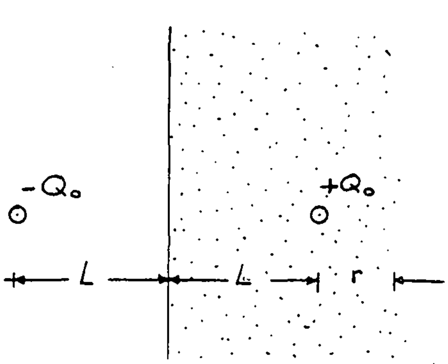
$$(13.33)^2-(12.80)^2 = \frac{(30)10^{-3}}{\pi\ k} \ln \frac{50}{20}$$

$$k = \frac{(30)10^{-3}(0.916)}{\pi(0.53)(26.13)} = (0.63)10^{-3} \text{ m/sec}$$

A more sophisticated approach takes into account the variation of R with distance to the shoreline. Using the method of images

$$H^2 - h^2 = \frac{Q_o}{\pi k} \ln \frac{2L + r}{r} \quad \text{and}$$

$$h_2^2-h_1^2 = \frac{Q_o}{\pi k} \ln \left(\frac{2L + r_1}{2L + r_2} \frac{r_2}{r_1}\right)$$



Provisionally assuming L = 400 m gives

$$(13.33)^2 - (12.80)^2 = \frac{(30)10^{-3}}{\pi k} \ln \frac{820}{850} \cdot \frac{50}{20}$$

$$k = \frac{(30)10^{-3}(0.88)}{\pi(0.53)(26.13)} = (0.61)10^{-3} \text{ m/sec}$$

The assumed value of L may be checked with the individual drawdowns

$$(15)^2 - (12.80)^2 = \frac{(30)10^{-3}}{\pi(0.61)10^{-3}} \ln \frac{2L + 20}{20}, \quad \ln \frac{2L + 20}{20} = 3.89 \quad L = 480\text{m}$$

$$(15)^2 - (13.33)^2 = \frac{(30)10^{-3}}{\pi(0.61)10^{-3}} \ln \frac{2L + 50}{50}, \quad \ln \frac{2L + 50}{50} = 3.00 \quad L = 480\text{m}$$

This gives finally

$$(13.33)^2 - (12.80)^2 = \frac{(30)10^{-3}}{\pi k} \ln \frac{980}{1010} \frac{50}{20}$$

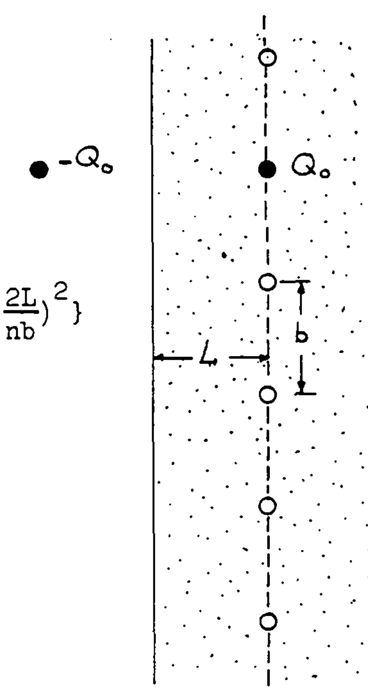
$$k = \frac{(30)10^{-3}(0.885)}{\pi(0.53)(26.13)} = (0.61)10^{-3} \text{ m/sec}$$

the same value as found above.

7.12 In an unconfined aquifer with a saturated thickness of 20 m above an impervious base, a line of fully penetrating wells runs at a distance of 50 m parallel to a stream. The wells have outside diameters of 0.6 m, an artificial gravel pack 0.15 m thick, intervals of 40 m and are pumped at constant rates of $(15)10^{-3}$ m^3/sec each. The wells are about 15 years old, but during the last years a serious lowering of the water level inside the pumped wells has been noticed, pointing to a clogging of the well screen openings. According to local experience, cleaning of the well screen is possible, but the cost is rather high and it is therefore doubtful whether this is an economic proposition. To decide this question, a test pumping is carried out, abstracting from one well an amount of $(28)10^{-3}$ m^3/sec and measuring the drawdown in the well itself and in the neighbouring wells

r =	0	40	80	120	160	200 m
s =	6.3	0.65	0.33	0.20	0.12	0.08 m

What is the amount of screen resistance and would cleaning of the well be an attractive proposition?



Using the method of images, the drawdown in a well at a distance nb from the pumped well equals

$$s = \frac{Q_o}{2\pi kH} \ln \frac{\sqrt{(nb)^2+(2L)^2}}{nb} = \frac{Q_o}{4\pi kH} \ln \{1+(\frac{2L}{nb})^2\}$$

This means that when s on linear scale is plotted against the factor

$a = 1 + (\frac{2L}{nb})^2$ on logarithmic scale,
a straight line will emerge, the slope of which is a measure for the coefficient of transmissibility kH. Unfortunately, however, the distance L is not known exactly. Indeed the wells are set at a distance of 50 m from the shoreline, but the effective distance to the constant water level in the bounding ditch might be larger. Two distances will therefore be assumed

$$L = 50 \text{ m}, \quad a_{50} = 1 + \left(\frac{100}{n\ 40}\right)^2 = 1 + \left(\frac{2.5}{n}\right)^2$$

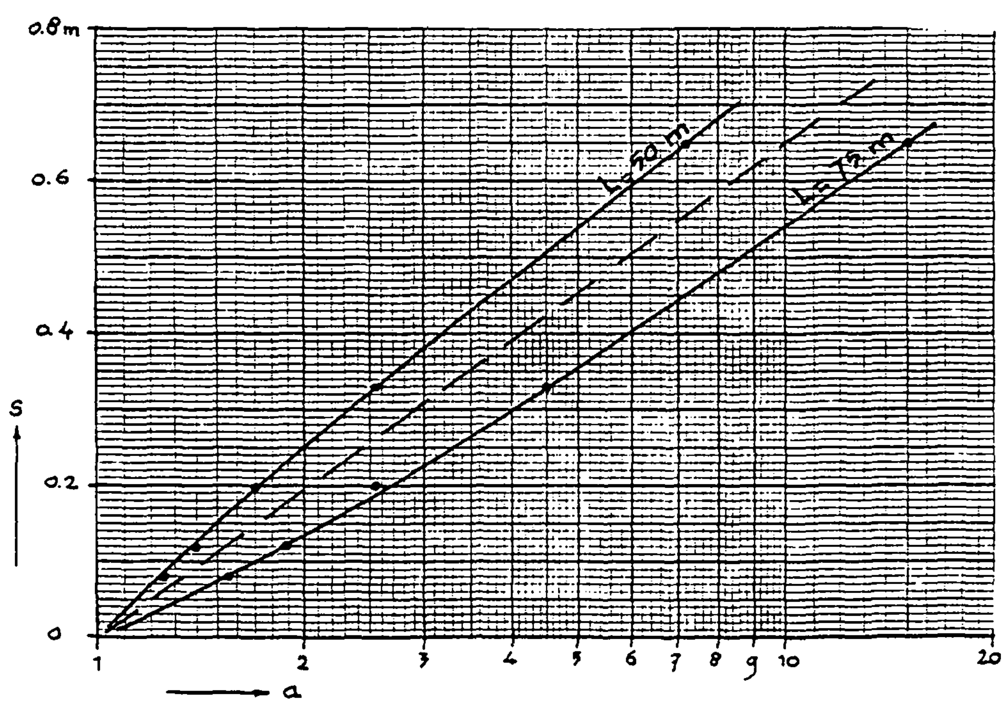
$$L = 75 \text{ m}, \quad a_{75} = 1 + \left(\frac{150}{n\ 40}\right)^2 = 1 + \left(\frac{3.75}{n}\right)^2$$

This gives

r =	40	80	120	160	200	m
n =	1	2	3	4	5	
a_{50} =	7.25	2.56	1.69	1.39	1.25	
a_{75} =	15.1	4.52	2.56	1.88	1.56	

Graphically the relation between s and a is shown in the diagram below, from which follows that L must be larger than 50 and smaller than 75 m. The dotted line has the average slope and as characteristics

$s = 0.65$ m, $r = 40$ m, $n = 1$, $a = 10$ from which follows



$$10 = 1 + (\frac{2L}{40})^2 \qquad \frac{2L}{40} = 3, \qquad L = 60 \text{ m}$$

$$0.65 = \frac{(28)10^{-3}}{4\pi kH} \ln 10, \quad kH = \frac{(28)10^{-3}(2.30)}{4\pi(0.65)} = (7.9)10^{-3} \text{ m}^2/\text{sec}$$

and with

$$H = 20 \text{ m}, \qquad k = (0.4)10^{-3} \text{ m/sec}$$

With these data, the drawdown at the well face follows from

$$H^2 - h_o{}^2 = \frac{Q_o}{\pi k} \ln \frac{2L}{r_o} \quad \text{or } 400 - h_o{}^2 = \frac{(28)10^{-3}}{\pi(0.4)10^{-3}} \ln \frac{120}{0.3}$$

$$h_o{}^2 = 400 - 134 = 266, \; h_o = 16.3 \text{ m}, \qquad s_o = 20 - 16.3 = 3.7 \text{ m}$$

In reality, a drawdown inside the well of 6.3 m was measured. The difference of 6.3 - 3.7 = 2.6 m is due to

entrance resistance

friction losses in well screen and casing pipe

With a flow of $(28)10^{-3}$ m^3/sec through screen and casing pipe of 0.3 m inner diameter, the velocity of upward watermovement is only 0.4 m/sec and even when the interior surface is very rough, the friction losses will not exceed 1 mm per m. With the shallow well under consideration, these friction losses are consequently negligeable and the full difference of 2.6 m must be attributed to entrance resistance.

During normal operation with a capacity of $(15)10^{-3}$ m^3/sec these entrance losses are much lower, equal to

$$\Delta = \{\frac{(15)10^{-3}}{(28)10^{-3}}\}^2 \; 2.6 = 0.75 \text{ m}$$

Theoretically this increases power requirements by

$$P = (15)10^{-3}(0.75) = (11)10^{-3} \text{ tonmeter/sec} = 0.11 \text{ kW}$$

and taking into account an over-all efficiency of 0.7 in reality by

$$P = \frac{0.11}{0.7} = 0.16 \text{ kW}$$

When operating continuously, the additional annual energy consumption equals

$$E = (0.16)(8760) = 1400 \text{ kWh}$$

increasing the cost of operation by about \$ 30 per year. This sum is so small, that an expensive cleaning operation is economically not justified.

7.21 In an unconfined aquifer of infinite extent, situated above an impervious base, a test well is pumped at a constant rate of $(12)10^{-3}$ m^3/sec. The resulting drawdown of the water table is measured with piezometers at distances of 20 and 50 m from the well centre. After 60 days of pumping these drawdowns are 1.65 m and 1.15 m respectively.

What are the values of the coefficient of transmissibility kH and of the specific yield μ for this formation?

The unsteady drawdown due to pumping a well in an unconfined aquifer above an impervious base equals

$$s = \frac{Q_o}{4\pi kH} W(u^2), \quad \text{with } u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

When u^2 is small, less than 0.05, a 98% accurate approximation may be had with

$$s = \frac{Q_o}{2\pi kH} \ln \frac{0.75}{u} = \frac{Q_o}{2\pi kH} \ln 1.5 \sqrt{\frac{kH}{\mu}} \frac{\sqrt{t}}{r}$$

Provisionally assuming that for the observations made u^2 is indeed small, the difference in drawdown in points at distances r_1 and r_2 from the well centre becomes

$$s_1 - s_2 = \frac{Q_o}{2\pi kH} \ln \frac{r_2}{r_1} \quad \text{or}$$

$$1.65-1.15 = \frac{(12)10^{-3}}{2\pi kH} \ln \frac{50}{20}$$

$$kH = \frac{(12)10^{-3}(0.916)}{2\pi(0.50)} = (3.5)10^{-3} \text{ m}^2/\text{sec}$$

The value of μ follows from (60 days = $(5.18)10^6$ sec)

$$s_1 = \frac{Q_o}{2\pi kH} \ln 1.5 \sqrt{\frac{kH}{\mu}} \frac{\sqrt{t}}{r_1}$$

$$1.65 = \frac{(12)10^{-3}}{2\pi(3.5)10^{-3}} \ln \quad 1.5 \sqrt{\frac{(3.5)10^{-3}}{\mu}} \quad \frac{\sqrt{(5.18)10^6}}{20}$$

$$\ln \frac{10.1}{\sqrt{\mu}} = 3.02 \ , \quad \frac{10.1}{\sqrt{\mu}} = 20.5 \ , \quad \mu = 0.24$$

To check the validity of the approximative formule, the largest value of u^2 used must be calculated

$$u_2^{\ 2} = \frac{0.24}{(4)(3.5)10^{-3}} \ \frac{(50)^2}{(5.18)10^6} = 0.0083 < 0.05$$

7.22 In an unconfined aquifer above an impervious base a fully penetrating well is pumped for 10 days at a constant rate Q_o of $(11)10^{-3}$ m^3/sec. The resulting lowering s of the phreatic water table is measured with piezometers at 20, 50 and 100 m from the well centre:

after	drawdown at distance of			
	20 m	50 m	100 m	$s_{20} - s_{50}$
1 day	0.46 m	0.13 m	0.02 m	0.33
2 days	0.62 m	0.22 m	0.04 m	0.40
3 "	0.73 m	0.30 m	0.07 m	0.43
5 "	0.85 m	0.42 m	0.15 m	0.43
7 "	0.91 m	0.48 m	0.20 m	0.43
10 "	1.00 m	0.56 m	0.26 m	0.44

What is the value of the coefficient of transmissibility kH and of the specific yield μ for this formation?

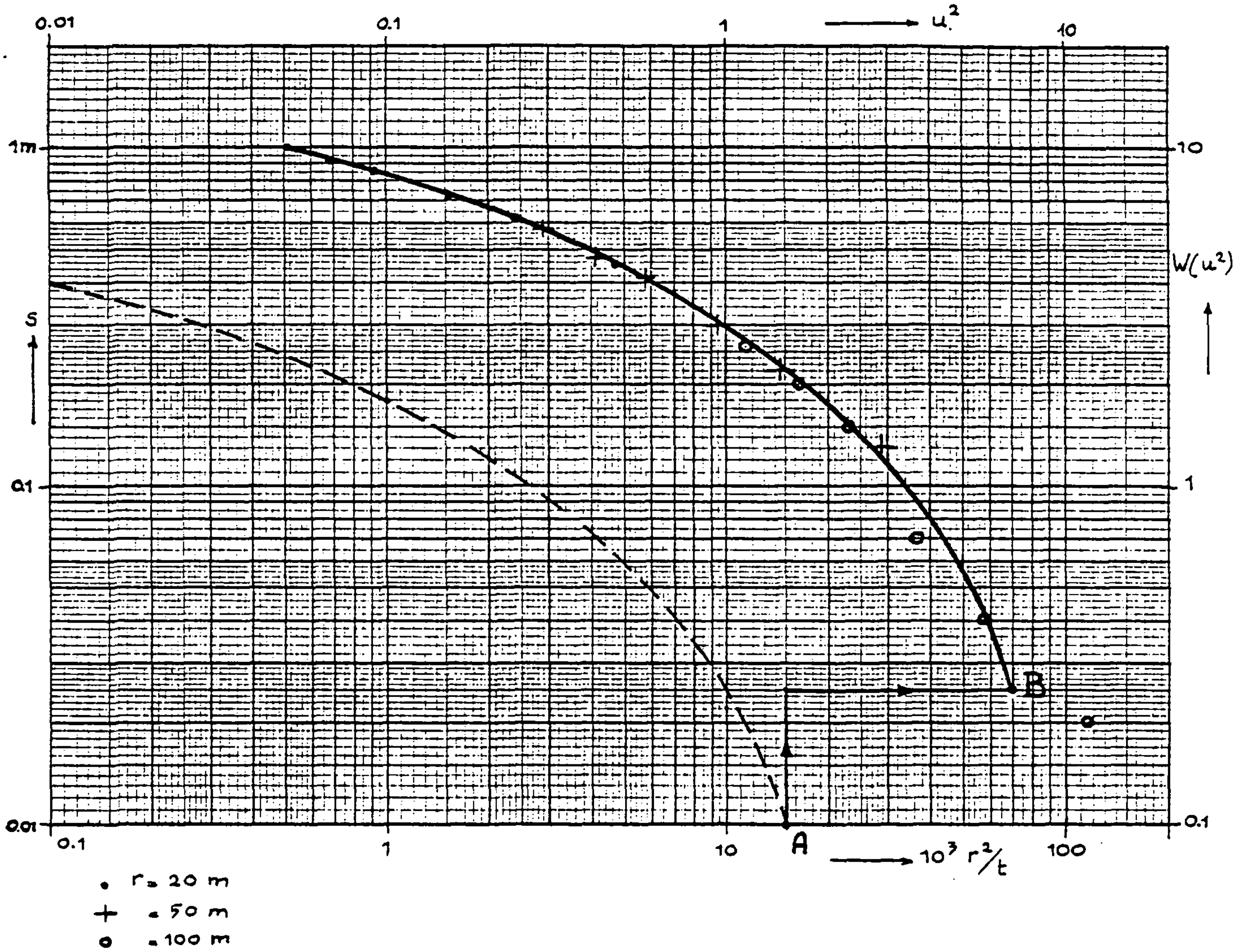
The unsteady drawdown in an unconfined aquifer above an impervious base equals

$$s = \frac{Q_o}{4\pi kH} W(u^2) \qquad \text{with}$$

$$u^2 = \frac{\mu}{4kH} \frac{r^2}{t}$$

For the various observations, the values of $\frac{r^2}{t}$ are tabulated below (1 day = 86400 sec)

after	r^2/t at distance of		
	20 m	50 m	100 m
1 day	$(4.63)10^{-3}$	$(28.9)10^{-3}$	$(116)10^{-3}$
2 days	(2.32)	(14.5)	(57.9)
3 "	(1.54)	(9.65)	(38.6)
5 "	(0.926)	(5.79)	(23.2)
7 "	(0.661)	(4.13)	(16.5)
10 "	(0.463)	(2.89)	(11.6)



In the accompanying diagram with a logarithmic division on both axis the drawdown s is plotted against the calculated values of r^2/t, while the logarithmic integral $W(u^2)$ is indicated with a dotted line. To cover the plotted points as well as possible, the dotted line must be moved upward and sideways, so that point A arrives in point B. The coordinates of both points

A: $u^2 = 1.5$ $\qquad W(u^2) = 0.1$

B: $\frac{r^2}{t} = (70)10^{-3}$ $\qquad s = 0.025$

must now correspond, giving as relations

$$0.025 = \frac{(11)10^{-3}}{4\pi kH} 0.1 , \qquad kH = (3.5)10^{-3}\ m^2/sec$$

$$1.5 = \frac{\mu}{(4)(3.5)10^{-3}} (70)10^{-3}, \qquad \mu = 0.30$$

The first table (page 7.22-a) shows, that the limit of $s_1 - s_2$ will be about 0.45 m.

Thiem's formula:

$s_1 - s_2 = \frac{Q_o}{2\pi kH} \ln \frac{r_2}{r_1}$ and substitution of this value and the data of $r_1 = 20$ m and $r_2 = 50$ m gives:

$$0.45 = \frac{(11)10^{-3}}{2\pi kH} \ln \frac{50}{20} \text{ or } kH = \frac{(11)10^{-3}\ 0.916}{2\pi\ (0.45)} = (3.6)10^{-3}\ m^2/sec$$

This result is in good accordance with the given solution.

7.23 Starting at t = 0, a well in an unconfined sandstone aquifer is pumped at a constant rate of 44 liters/sec. In an observation well at a distance of 75 m the drawdown equals

t = 100	1000	10000	minutes
s = 0.57	1.01	1.44	m

What are the values for the coefficient of transmissibility and for the specific yield of this aquifer?

The drawdown accompanying the unsteady flow of groundwater to a well is given by

$$s = \frac{Q_o}{4\pi kH} W(u^2) \quad \text{with } u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

For u^2 small, that is for t large, this formula may be simplified to

$$s = \frac{Q_o}{4\pi kH} \ln \frac{0.562}{u^2}$$ and after substitution of the value for u^2

$$s = \frac{Q_o}{4\pi kH} \ln \frac{(2.25)kH}{\mu r^2} t$$

With the observations made, this gives

$$(1) \quad 0.57 = \frac{(44)10^{-3}}{4\pi kH} \ln \frac{(2.25)kH}{\mu(75)^2} (100)(60)$$

$$(2) \quad 1.01 = \frac{(44)10^{-3}}{4\pi kH} \ln \frac{(2.25)kH}{\mu(75)^2} (1000)(60)$$

$$(3) \quad 1.44 = \frac{(44)10^{-3}}{4\pi kH} \ln \frac{(2.25)kH}{\mu(75)^2} (10000)(60)$$

$$(1)-(2) \quad 0.44 = \frac{(44)10^{-3}}{4\pi kH} \ln 10$$

$$(2)-(3) \quad 0.43 = \frac{(44)10^{-3}}{4\pi kH} \ln 10$$

$$(1)-(3) \quad 0.87 = \frac{(44)10^{-3}}{4\pi kH} \ln 100 \quad \text{or } 0.435 = \frac{(44)10^{-3}}{4\pi kH} \ln 10$$

These results are in good accordance with each other, justifying the use of the approximate formula for $W(u^2)$. With the last equation the value of kH follows at

$$kH = \frac{(44)10^{-3}}{4\pi(0.435)} \ln 10 = 0.0185 \text{ m}^2/\text{sec} = 1600 \text{ m}^2/\text{day}$$

Substitution of this value in (3) gives

$$1.44 = \frac{(44)10^{-3}}{4\pi(0.0185)} \ln \frac{(2.25)(0.0185)}{\mu(75)^2} (10000)(60)$$

$$7.61 = \ln 2015 = \ln \frac{4.44}{\mu} \qquad \text{or } \mu = 0.0022 = 0.22\%$$

7.24 An unconfined fractured limestone aquifer has a saturated thickness of about 300 m and is situated above an impervious base. In this aquifer an open hole is drilled with a diameter of 0.24 m. Starting at t = 0 water is abstracted from this hole in a constant amount of 0.3 m^3/minute, giving as drawdown

t =	0	5	10	15	30	45	60	120	minutes
s_o =	0.0	21.5	24.0	25.5	28.0	29.5	30.5	33.0	m

Questions

a. what are the geo-hydrological constants of this aquifer?

b. what is the drawdown after 3 months of continuous pumping at the rate mentioned above?

c. what is the remaining drawdown 9 months after cessation of pumping?

The unsteady drawdown in an unconfined aquifer above an impervious base equals

$$s = \frac{Q_o}{4\pi kH} W(u^2) \text{ with } u^2 = \frac{\mu}{4kH}\frac{r^2}{t}$$

At the well face, $r = r_o$, u^2 will be small, allowing as approximation

$$s_o = \frac{Q_o}{4\pi kH} \ln \frac{0.562}{u^2} = \frac{Q_o}{4\pi kH} \ln \frac{2.25\ kH}{\mu r_o^2} t$$

The observation clearly show that each doubling of time increases the drawdown by 2.5 m. Substituted

$$2.5 = \frac{Q_o}{4\pi kH} \ln 2$$

$$kH = \frac{Q_o}{(10)\pi} \ln 2 = \frac{0.3}{(10)\pi(60)} \ln 2 = (0.11)10^{-3}\ m^2/sec$$

After 120 min = 7200 sec

$$33.0 = \frac{0.3}{(4)\pi(0.11)10^{-3}(60)} \ln \frac{(2.25)(0.11)10^{-3}}{\mu(0.12)^2} 7200$$

$$33.0 = 3.617 \ln \frac{123.8}{\mu}, \quad \ln \frac{123.8}{\mu} = 9.124 = \ln 9169$$

$$\mu = \frac{123.8}{9169} = 0.0135 = 1.35\%$$

After 3 months = $(7.884)10^6$ sec the drawdown becomes

$$s_o = 3.617 \ln \frac{(2.25)(0.11)10^{-3}}{(0.0135)(0.12)^2} (7.884)10^6 =$$

$$= 3.617 \ln (10.04)10^6 = 58.3\ m$$

Nine months later the remaining drawdown equals

$$s_o = 3.617 \ln \frac{(2.25)(0.11)10^{-3}}{(0.0135)(0.12)^2}(4)(7.884)10^6 -$$

$$- \ln \frac{(2.25)(0.11)10^{-3}}{(0.0135)(0.12)^2}(3)(7.884)10^6$$

$$s_o = 3.617 \ln \frac{4}{3} = 1.04 \text{ m}$$

Strictly speaking, the drawdown of 58.3 m decreases the saturated thickness to 300 - 58 = 242 m or to $\frac{242}{300}$ = 0.8 of the original value. This increases the drawdown to

$$s_o = \frac{3.617}{0.8} \ln (10.04)10^6(0.8) = 71.9 \text{ m or by } 23\%$$

and so on, and so on. By trial and error

$$s_o = 75 \text{ m} \quad \frac{H - s_o}{H} = \frac{225}{300} = 0.75$$

$$s_o = \frac{3.617}{0.75} \ln (10.04)10^6(0.75) = 76.4 \text{ m}$$

$$s_o = 80 \text{ m} \quad \frac{H - s_o}{H} = \frac{220}{300} = 0.733$$

$$s_o = \frac{3.617}{0.733} \ln (10.04)10^6(0.733) = 78.0$$

from which follows by interpolation s_o = 77.1 m

8.01 How are round holes obtained when drilling with the cable tool percussion method in consolidated formations and what is the purpose of the sinker bar in this system?

With the cable tool percussion method of well construction, the tool string is supported in the drill hole at the end of a steel cable, connected to it by means of a rope socket. The cable is woven such as to obtain a strong twist, while the rope socket allows the tool string to turn relative to the cable. When now during the up-stroke the cable stretches, its twist will turn the tool-string several times. At the end of the downstroke, the tool-string rests momentarily upon the bottom of the hole, the cable slackens and is now able to turn back by the swivel action of the rope socket. The actual amount of turning, however, is never the same in both directions. This assures that the drill bit will strike the bottom of the hole every time in a different position, thus producing round holes.

When by small cave-ins for instance, the tool string sticks in the drill hole, straight pulling is commonly insufficient to loosen it and the tool must be hammered to above. This can be done with the drill rig, after the cable has been lowered till the set of jars is completely closed. The necessary force for this operation is now provided by the weight of the sinker bar.

8.02 What is the purpose of the set of jars in the cable tool percussion method of well drilling and why is the length of the auger stem so great?

The set of jars provide a loose link in the tool string. When now during drilling the bit strikes the bottom of the hole before the end of the downstroke, this set of jars closes, taking up the slack of the cable, thus preventing bending and rapid breaking of the cable. During the upstroke the set of jars allow the sinker bar on top to give a sharp upward blow to the tool string, preventing it from sticking or wedging in the hole.

A great length of auger stem is required to obtain straight and vertical holes, especially important when for groundwater abstraction well pumps must be used, which are set inside the casing at some depth below the water level during operation.

8.03 What is the difference between straight and reverse hydraulic well drilling?

With hydraulic well drilling, a flow of water is used for continuous removal of drill cuttings. With straight hydraulic well drilling, this flow is directed down the hollow drill pipe and flows upward in the annular space between the drill pipe and the drill hole. With reverse hydraulic well drilling, the circulating fluid flows downward in the annular space mentioned above and rises inside the drill pipe. With the latter method, the cutting action of the rotating drill tools is no longer supported by the jetting action of the stream of water. Inside the drill pipe, however, the upflow velocities are now much higher, enabling large chunks of material to be carried to ground surface.

8.11 Which method of gravel placement around a well can best be applied when a single and when a triple gravel treatment is to be used?

With a single gravel treatment, the screen is centered in the drill hole by means of guide blocks and the gravel is poured into the remaining annular space. To prevent de-segregation during the fall through water, small diameter filling pipes can be used with which the gravel can be fed in slowly and evenly. During placement of the gravel, the casing and filling pipes are slowly raised, keeping the bottom of the casing 1 or 2 m below the top of the gravel and the lower end of the filling pipes not more than 0.5 m above this top.

With a triple gravel treatment, the bottom of the well screen is enlarged with a heavy wooden disk, on to which wire gauze packing baskets are fastened. The two innermost layers of gravel are now filled in above ground, allowing carefull inspection, equal wall thicknesses and a stable packing. After the screen with the attached layers of gravel has been lowered and centered in the hole, the remaining annular space is filled with the finest gravel, in the same way as described above for a single gravel treatment.

8.12 Why is it desirable to keep the top of the well screen some distance below the lowest groundwater level during operation?

Every water table well abstracts two types of water

1. water that has infiltrated directly around the well
2. water that has infiltrated some distance away from the well.

Directly below the water table, the groundwater type 1 is aerobic. In case the sub-soil contains organic matter, the water type 2, however, is anaerobic and may have picked up ferrous iron from the underground. Around the well screen, both types of water will mix, converting the soluble ferrous iron into insoluble ferrix oxyde hydrates which clog the well screen openings

$$2Fe^{++} + O_2 + (n + 1)H_2O \rightarrow Fe_2O_3.n(H_2O) + 2H^+$$

To prevent this phenomenon from occurring, the water type 1 must also be anaerobic. This can be obtained by forcing this water to travel a greater distance through the sub-soil, by keeping the well screen openings some distance below the lowest water level during operation.

8.13 In fine formations, gravel treatment is used to allow the use of screens with larger openings. What is the maximum permissible increase in slot width when a single gravel layer is applied?

With an artificial gravel pack, the slot width b of the screen openings must be 2 to 3 times smaller than the lower grain size limit of the inner gravel layer, while with a single gravel treatment this grain size limit must be smaller than 4 times the 85% diameter of the aquifer material. The maximum slot width thus becomes

$$b = \frac{1}{2 \text{ to } 3} \, 4 \, d_{85} = (1.3 \text{ to } 2) d_{85}$$

Without artificial gravel treatment and non-uniform aquifer material the well screen openings may pass 80% of the aquifer material

$$b' = d_{80} \simeq 0.95 \, d_{85}$$

With uniform material the percentage of aquifer material passing must be reduced to 40%

$$b' = d_{40} \simeq 0.8 \, d_{85}$$ This gives

$$\frac{b}{b'} = \frac{(1.3 \text{ to } 2) d_{85}}{(0.8 \text{ to } 0.95) d_{85}} = 1.4 \text{ to } 2.5$$

8.21 From a well water is abstracted with a submersible pump. This pump, however, must be set inside the well screen. Which provisions are now advisable?

Around the pump inlet large velocities of flow will occur, which are even able to shift the material outside the screen. On the long run this will result in a destruction of the screen by continuous friction with the grains. To prevent such damage, the screen must be replaced by a blank piece, about 1 m long, around the pump inlet.