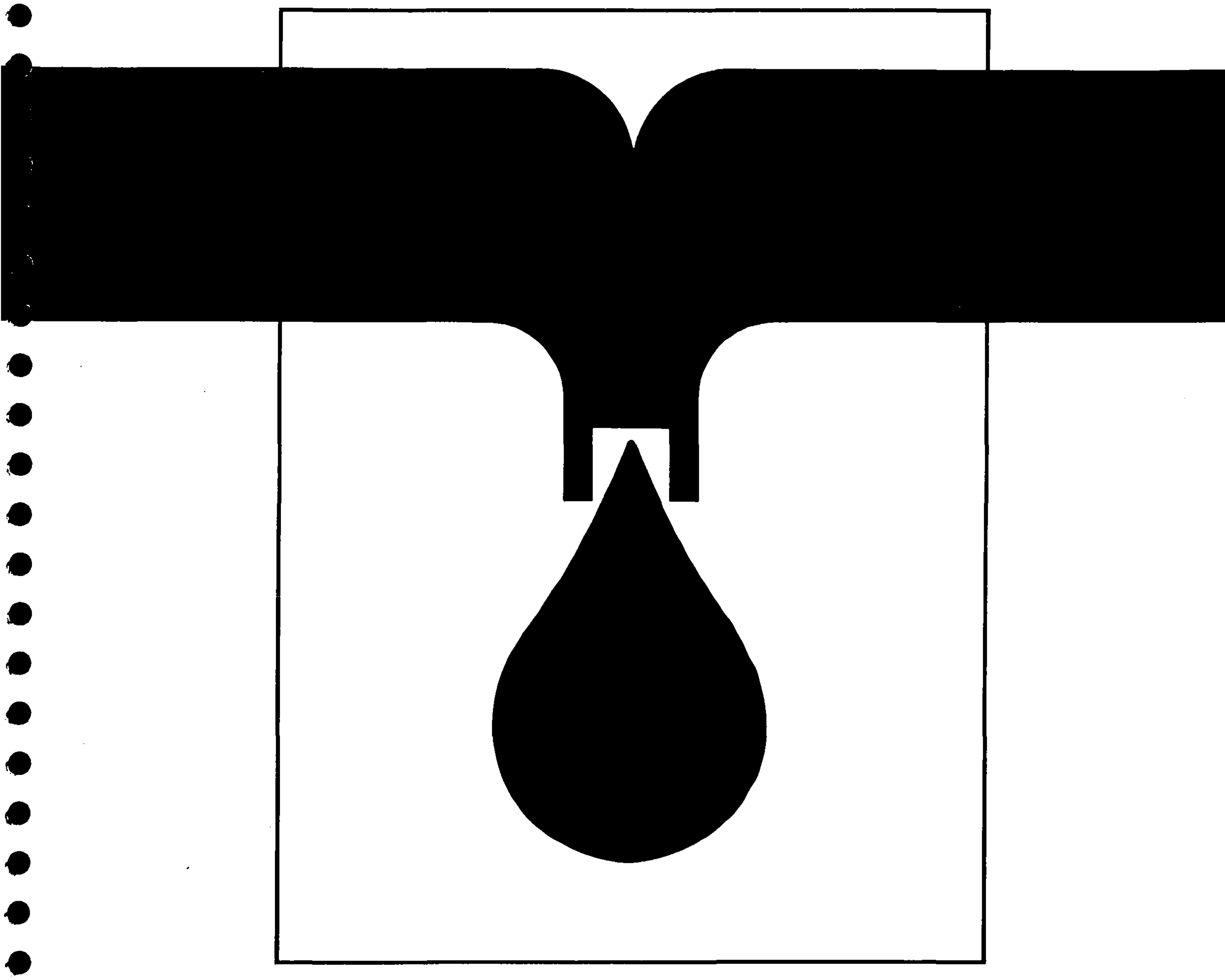
TRAINING MODULES
FOR
WATERWORKS PERSONNEL



Basic Knowledge

0.1

Basic and applied arithmetic

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Foreword

Even the greatest optimists are no longer sure that the goals of the UN "International Drinking Water Supply and Sanitation Decade", set in 1977 in Mar del Plata, can be achieved by 1990. High population growth in the Third World combined with stagnating financial and personnel resources have led to modifications to the strategies in cooperation with developing countries. A reorientation process has commenced which can be characterized by the following catchwords:

- use of appropriate, simple and - if possible - low-cost technologies,
- lowering of excessively high water-supply and disposal standards,
- priority to optimal operation and maintenance, rather than new investments,
- emphasis on institution-building and human resources development.

Our training modules are an effort to translate the last two strategies into practice. Experience has shown that a standardized training system for waterworks personnel in developing countries does not meet our partners' varying individual needs. But to prepare specific documents for each new project or compile them anew from existing materials on hand cannot be justified from the economic viewpoint. We have therefore opted for a flexible system of training modules which can be combined to suit the situation and needs of the target group in each case, and thus put existing personnel in a position to optimally maintain and operate the plant.

The modules will primarily be used as guidelines and basic training aids by GTZ staff and GTZ consultants in institution-building and operation and maintenance projects. In the medium term, however, they could be used by local instructors, trainers, plant managers and operating personnel in their daily work, as check lists and working instructions.

45 modules are presently available, each covering subject-specific knowledge and skills required in individual areas of waterworks operations, preventive maintenance and repair. Different combinations of modules will be required for classroom work, exercises, and practical application, to suit in each case the type of project, size of plant and the previous qualifications and practical experience of potential users.

Practical day-to-day use will of course generate hints on how to supplement or modify the texts. In other words: this edition is by no means a finalized version. We hope to receive your critical comments on the modules so that they can be optimized over the course of time.

Our grateful thanks are due to

Prof. Dr.-Ing. H. P. Haug
and
Ing.-Grad. H. Hack

for their committed coordination work and also to the following co-authors for preparing the modules:

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It is my sincere wish that these training modules will be put to successful use and will thus support world-wide efforts in improving water supply and raising living standards.

Dr. Ing. Klaus Erbel
Head of Division
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Eschborn, May 1987

Basic and applied arithmetic

Table of contents:

1 The basic arithmetical operations - nomenclature and signs

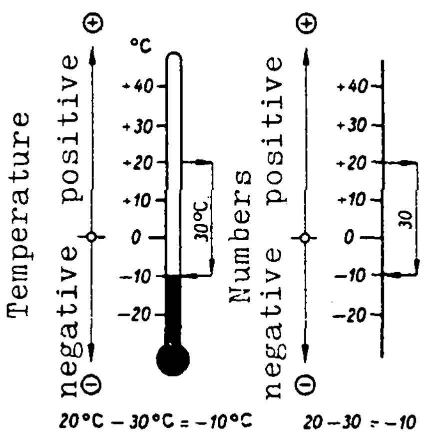
Operation	Sign	Operation	Sign
Addition	+	Multiplication	x
Subtraction	-	Division	:

Addition				
Addend	+	augend	=	sum
3	+	2	=	5

Subtraction				
Minuend	-	subtrahend	=	difference (remainder)
5	-	2	=	3

1.1 Positive and negative numbers

On an thermometer, the temperature scale continues below the point zero. If the temperature in a refrigerator drops from + 20°C by 30°, the thermometer then shows the final temperature as -10°C. Similarly, in mathematics it is also possible to operate with numbers below the value zero. Thus a larger figure can be subtracted from a smaller one.



A distinction is made between positive and negative numbers. The sign in front of a positive number is often omitted, e.g. +10 = 10. If there is no sign in front of a number, a "plus" sign should be assumed i.e. 10 = +10. Negative numbers are, for instance, -5, -0.2, -3/8.

1.2. Working with positive and negative numbers

Example: 180 - 26 + 75 - 3 = 180 + 75 - 26 - 3

255 - 29 = 226

90 - 135 + 15 - 7 = 90 + 15 - 135 - 7

105 - 142 = - 37

First of all the sum of the positive and the sum of the negative terms is found. Then the sum of the negative terms is subtracted from the sum of the positive terms.

1.3. Multiplication

Basic concepts:

Multiplier	times	multiplicand	=	product
(1st) factor		(2nd) factor		
3	x	4	=	12

Factors may be multiplied by each other in any order, e.g.
3 x 5 = 5 x 3; 6 x 8 x 7 = 7 x 6 x 8

Where large numbers are involved, the calculation is worked out on paper and is known as long multiplication.

Example: 6152 x 7 (one factor is a single digit)

(6000 + 100 + 50 + 2) x 7 = 6000 x 7 + 100 x 7 + 50 x 7 + 2 x 7

```
= 42000
+   700
+   350
+    14
-------
  43064
```

In practice, this process is carried out by an abbreviated method in the order shown by the arrow.

```
6 1 5 2 x 7
-----------
      43064
```

The end digits of the partial products (e.g. 7 x 2 = 14) are written down from right to left, beginning underneath the multiplier (7). The remaining digit is added to the next partial product, thus 5 x 7 + 1 = 36.
The same principle applies to multiplications with end

noughts. The calculation is carried out a described and the noughts added on the far right.

Example:

```
3168  x 3000
     9504000
```

To multiply factors which both have more than one digit, the multiplier can be resolved, e.g. 631 x 354 is calculated by resolving into 631 x (300 + 50 + 4).

```
631 x 354                       631 x 354
631 x 300 =      189300         631 x 3 =     1893
631 x  50 = +     31550         631 x 5 =      3155
631 x   4 = +      2524         631 x 4 =       2524
                 223374                      223374
```

When this operation has been fully mastered, the noughts can be omitted (as shown on the right).
With enough practice, resolution of the numbers as shown on the left can also be eliminated, so that the following scheme results:

```
631  x 354
   1894
    3155
     2524
   223374
```

In long multiplication, it is essential that the columns of figures should begin directly underneath the appropriate digit of the multiplier. The row then continues from right to left in such a way that each digit comes exactly underneath the one above. It is useful when starting to use squared paper. Care should be taken when there are noughts in the right-hand factor (multiplier). These are taken into account by appending them to the row above, e.g.

```
15245  x  12050
       15245
        304900
         762250
       183702250
```

Rule of sign in multiplication

If 2 factors have the same sign, the product is positive.

(+3) x (+5) = +15: + times + make +

(-3) x (-5) = +15: - times - make +

Like signs give a positive product.

If 2 factors have different signs, the product is negative.

(-3) x (+5) = -15: - times + make -

(+3) x (-5) = -15: + times - make -

Unlike signs give a negative product.

1.4 Division

The same rule of sign applies in division as in mulitplication. Like signs make +, unlike signs make -.

Dividend	divided by	divisor	=	quotient
48	:	8	=	6

Division is simple if the dividend is to be found in the multiplication table of the divisior and the divisor is not higher than 12. All such problems can be solved by mental arithmetic after some practice. This ability is essential for carrying out divisions of all kinds. Where larger numbers are involved, the problem has to be worked out on paper - or with ohter means - and is called long division.

Long division

The method is explained below using an example with a single-digit divisor.

5495 : 7 One after the other, the number of thousands, hundreds, tens and units in the answer are found, beginning with the highest possible value.

7 x 1000 is more than 5495, so there are no thousands.

To find the hundreds, the question is asked how many times 7 goes into 5495. The answer is 700, but not 800 times, so

```
                          5495 ÷  7 = 700  + ...
         - 7  x 700 =     4900
           Remainder       595
```

Further, 7 goes into 595 80, but not 90 times, so

```
                          5495 ÷  7  =  700 + 80 + ...
                          4900
                           595
         -  7  x 80  =     560
              Remainder     35
```

7 goes into 35 exactly 5 times, thus

```
                          5495  ÷  7 =  700 + 80 + 5
                          4900
                           595
                           560
                            35
         - 7  x  5  =       35
                             0
```

Thus 5495 ÷ 7 = 785; there is no remainder. In practice, the operation is abbreviated and the noughts omitted.

```
Example:       5495  ÷  7  =  785    54  ÷  7 gives = 7
Intermed- ——▸ 49                     7  x 7 = 49, remainder 5
iate product   59                    59 ÷ 7 gives    = 8
                56                   8  x 7 = 56, remainder 3
                 35                  35 ÷ 7 gives    = 5
                 35                  5 x 7 = 35, remainder 0
                   0
```

The numbers underlined twice are written in the answer on the right of the equals sign (=). The remainders, underlined

once, result from subtraction of the intermediate products. When these have been written down, the next digit of the dividend is brought down in the correct position.
The same method is applied in principle when the divisor has more than one digit.
Special care must be taken where there are end noughts in the answer and at the end of the dividend. Consider the following examples:

1.

```
221996 ÷ 437 = 508
2185
  3496
```

349 is smaller than 437; there are no tens in the answer. We therefore write down 0, then bring down the next digit (6) and continue in the usual way.

2.

```
51836638 ÷ 1234 = 42007
4936
 2476
 2468
   8638
```

86 ÷ 1234 gives 0; 3 down. 836 : 1234 gives 0 8 down. Continue in the usual way.

3.

```
13952000 ÷ 545 = 25600
1090
 3052
 2725
  3270
  3270
     000
```

Following the answer 6, two more noughts have to be brought down in succession. 545 goes into both 00 and 000 0 times, however, therefore there are two noughts at the end of the answer.

Division with remainders
In long division, the remainder automatically appears at the end of the calculation, e.g.

```
3635 ÷ 314 = 11 remainder 181
314                       ---
 495                      314
 314
  181
```

1.5 Order of operations

If we consider the expression 3 x 4 + 5, we notice that this could be interpreted in more than one way. Taking the figures in the order in which they appear, 3 x 4 = 12; 12 + 5 = 17; if written by the bracket method (whereby figures in brackets are worked out first), this would appear as (3 x 4) + 5. By moving the brackets, we can also write, instead of (3 x 4) + 5, 3 x (4 + 5), i.e. 4 + 5 = 9; 3 x 9 = 27.

Thus the result is not the same as in the first example. To avoid confusion, it has been agreed that expressions such as 3 x 4 + 5, 20 - 8 x 2, 17 + 3 x 12 - 15 should be read as follows:

(3 x 4) - 5, 20 - (8 x 2), 17 + (3 x 12) - 15, i.e. the rule is:

multiplication or division (x and ÷)
come before
addition or subtraction (+ and -)

Thus 3 x 4 + 5 = 17; 20 - 8 x 2 = 4; 17 + 3 x 12 - 15 = 38.

2 Fractions

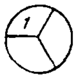
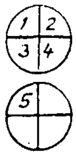
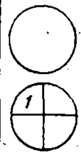
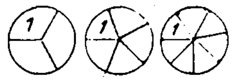
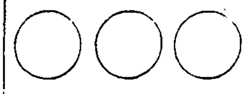
2.1 General points

If 4 children want to share an apple equally, this can only be done if the apple is cut into 4 equal parts and each child receives one-fourth of the apple. Since 4 is not a divisor of 1, this division would not normally work out evenly. A solution is possible, however, if we write the answer to the problem $\frac{1}{4}$. Thus the whole apple consists of $\frac{4}{4}$.

The figure above the horizontal line (sign of division) gives the number of parts while the figure below the line names the parts.

Example: 3 → numerator
4 → denominator

2.2 Types of fractions

Proper Fraction 1	Improper Fraction 1	Mixed Number	Fractions with a common denominator	Fractions without a common denominator	False Fraction
 $\frac{1}{3}$	 $\frac{5}{4}$	 $1\frac{1}{4}$	 $\frac{1}{3}$ $\frac{3}{8}$ $\frac{5}{8}$	 $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{7}$	 $\frac{3}{1} = 3$
Numerator 1 smaller than denominator 3	Numerator 5 larger than denominator 4	Integer with fraction	Denominator the same	Denominator different	Denominator equals 1

2.3. Modification of fractions

Expansion

Since any number multiplied by 1 retains its value (5 x 1 = 5) and e.g. $\frac{3}{3} = 1$, the numerator and denominator of any fraction may both be multiplied by the same number. This is called expansion.

$\frac{1 \ (x\ 2)}{2 \ (x\ 2)} = \frac{2}{8}$	Expansion means that numerator and denominator are both multiplied by the same number. The value of the fraction remains the same.

Examples: give the value of x.

1.) $\frac{2}{3} = \frac{x}{9}$ 2.) $\frac{2}{5} = \frac{8}{x}$

In both examples, either the numerator or the denominator of an expanded fraction is to be found. First of all the expansion factor - i.e. the number by which numerator or denominator was multiplied - is found by comparison of the two numerators or denominators. This is then used to multiply the other half of the fraction.

1.) 9 ÷ 3 = 3. The denominator was multiplied by 3. Therefore the numerator must also be multiplied by 3:
x = 2 x 3 = 6.

2.) 8 ÷ 2 = 4. The numerator was multiplied by 4. Therefore the denominator must also be multiplied by 4:
x = 5 x 4 = 20.

Simplification

Since it is equally true that any number divided by 1 retains its value, it follows that numerator and denominator may be divided by the same number, since $\frac{4}{4} = 1$

$\frac{2\ (\div 2)}{8\ (\div 2)} = \frac{1}{4}$	Simplification means that nominator and denominator are divided by the same number. The fraction retains the same value but is simpler.

Examples: $\frac{27}{45} = \frac{x}{5}$

The denominator was divided by 9, therefore the numerator must also be divided by 9. Thus x = 27 ÷ 9 = 3.

$$\frac{18\ (\div 9)}{27\ (\div 9)} = \frac{2}{3}$$

To become proficient at expanding and simplifying fractions, it is not sufficient just to solve a few problems. Only repeated practice makes it possible to recognize the (greatest) common divisor quickly, so that the calculation can be continued after simplification with numbers which are smaller and thus easier to handle.

If the numerator or the denominator is given as a sum or a difference, this must be solved before proceeding to expand or simplify the fraction.

$$8 \times \frac{9 + 5}{280} = \frac{8 \times 14}{280} = \frac{8 \times 14}{20 \times 14} = \frac{2}{5}$$

$$\frac{4 - 3}{4} = \frac{1}{4}$$

2.4 Addition and subtraction of fractions

Fractions which have the same denominator are added or subtracted by adding or subtracting the numerators and leaving the denominator unaltered.

Where fractions have different denominators, the lowest common denominator must be found. This is the lowest number into which the denominators of all the fractions can be divided without remainder.

Determination of the lowest common denominator:

Example: $\frac{7}{12} + \frac{2}{45} - \frac{5}{18} = ?$

Solution: All denominators are split up into their smallest factors. Equal factors are left next to the numbers in which they occur most frequently. The others are cancelled out and all remaining factors multiplied to find the product, which is the lowest common denominator.

$12 = 2 \times 2 \times \not{3}$

$45 = \not{3} \times \not{3} \times 5$

$18 = 3 \times 3 \times \not{2}$

The remaining factors are: $2 \times 2 \times 3 \times 3 \times 5 = 180$ (lowest common denominator)

Following this, the expansion factors are determined by dividing the denominator of each fraction into the lowest common denominator.

In our example: $180 \div 12 = 15$

$180 \div 45 = 4$

$180 \div 18 = 10$

Once the numerators of all the fractions have been expanded by multiplying them with the factors thus determined, they can be added or subtracted:

$$\frac{7}{12} + \frac{2}{45} - \frac{5}{18} = \frac{15 \times 7}{180} + \frac{4 \times 2}{180} - \frac{10 \times 5}{180}$$

$$= \frac{105 + 8 - 50}{180} = \frac{63}{180} = \frac{\not{9} \times 7}{\not{9} \times 20} \quad \frac{7}{20}$$

2.5 Multiplication and division of fractions

Integer times a fraction:

$4 \times \frac{2}{3} = \frac{4}{1} \times \frac{2}{3} = \frac{8}{3} = 2\,\frac{2}{3}$

The numerator of the fraction is multiplied by the integer; the denominator remains unaltered.

Fraction times a fraction:

$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$

The numerator is multiplied by the numerator, the denominator by the denominator.

Mixed number times an integer:

$$2\frac{1}{3} \times 3 = \frac{7}{3} \times 3 = \frac{7 \times 3}{3} = \frac{21}{3} = 7$$

The mixed number is converted into an improper fraction and the numerator multiplied by the integer.

Mixed number times mixed number:

$$1\frac{3}{4} \times 3\frac{1}{2} = \frac{7}{4} \times \frac{7}{2} = \frac{49}{8} = 6\frac{1}{8}$$

The mixed numbers are first converted into impropber fractions, then numerator multiplied by numerator and denominator by denominator.

Fraction divided by an integer:

$$\frac{1}{4} : 3 = \frac{1}{4 \times 3} = \frac{1}{12}$$

The denominator is multiplied by the integer. The numerator stays the same.

Integer divided by a fraction:

$$5 \div \frac{3}{3} = \frac{5 \times 4}{3} = \frac{20}{3} = 6\frac{2}{3}$$

The integer is multiplied by the <u>reciprocal</u> of the fraction.

Fraction divided by a fraction (compound fraction):

$\frac{3}{4} \div \frac{3}{5} = ?$ Often a horizontal line is used instead of the sign of division (÷).

Numerator fraction

Denominator fraction

$$\frac{\frac{3}{4}}{\frac{3}{5}} = \frac{3 \times 5}{4 \times 3} = \frac{5}{4} = 1\frac{1}{4}$$

The numerator fraction is multiplied by the <u>reciprocal</u> of the denominator fraction.

2.6 Decimal fractions

Conversion of vulgar fraction into a decimal fraction:

$\frac{3}{8} = 3 \div 8 = 0.375$ The numerator is divided by the denominator.

Conversion of a finite decimal fraction into a vulgar fraction:

$0.48 = \frac{48}{100} = \frac{24}{50} = \frac{12}{25}$; $0.345 = \frac{345}{1000}$, $2.75 = 2\frac{75}{100} = 2\frac{3}{4}$

All the digist to the right of the decimal point are written as the numerator. The denominator hat a 1 and as many noughts as there are digits in the numerator.

3 Proportions

The method used in calculations of this kind is basically an application of the rules of fractional arithmetic. It is essential to be able to recognize whether the result of multiplying or of dividing one number by another is a larger or smaller number.

Examples $\frac{6}{2} = 3$	The result is smaller, i.e. less.
$\frac{6}{0.5} = 12$	The result is larger, i.e. more.
$4 \times 2 = 8$	The result is larger, i.e. more.
$4 \times 0.5 = 2$	The result is smaller, i.e. less.

3.1 Rule of three (unitary) method

Examples:

1. If 8 litres of milk cost 8.40 DM, how much do 13 litres cost?
2. If 4 workmen take 12 days to build a wall, how long would 6 workmen take to build the same wall?

Each conditional statement must be re-formulated so that it consists of a statement with the required dimension at the end and a question with the known dimension and the unknown quantity in the same positions as in the statement.

1st step: 8 litres cost 8.40 DM (statement, required dimension at the end)

13 litres cost ? DM (question, unknown quantity at the end)

Now the unit of the known quantity is determinded by asking the question: If 8 litres cost 8.40 DM, does 1 litre cost more or less than 8 litres? The answer ist less, so the DM price must be smaller, i.e. must be divided by 8.

2nd step: 1 litre thus costs $\frac{8.40 \text{ DM}}{8}$

In the third step, the unit of the known quantity is used to calculate the multiple. If 1 litre costs 8.40 DM : 8, do 13 litres cost more or less than 1 litre; The answer is more, so the price of 1 litre must be multiplied by 13.

3rd step: 13 litres cost $\frac{8.4 \times 13}{8} = 13.65$ DM

In the above example, the quantities are directly proportional: i.e. more gives a larger figure, less gives a smaller figure.

Example 2:

1st step:	4 workmen take 12 days	Statement
	6 workmen take ? days	Question

2nd step: 1 workman takes 12 x 4 days

It is advisable when proceeding to the 2nd step to ask the question: If 4 workmen take 12 days to carry out the work, does 1 workman need more or less days to complete the same work? Answer: he needs 4 times as long.

3rd step: 6 workmen take $\frac{12 \times 4}{6} = 8$ days

The 3rd step can be formulated as follows: If 1 workman takes 12 x 4 days, do 6 workmen need more or less days for the same work? Answer: they need one-sixth of the time.

In the second example, quantities are in inverse proportion, i.e. more gives a smaller figure, less gives a larger figure.

Complex rule-of-three problem

In a normal rule of three computation, 3 quantities are known and a 4th quantity is to be found; there are 2 dimensions. If there are more than 2 dimensions, this constitutes a complex rule-of-three problem.

To solve a complex rule-of-three problem, it is first divided into separate simple problems and the unit of each dimension in the statement calculated in succession.

Example:

If 5 men (M) take 12 days to build a wall 8 m long, working at a rate of 7 hours a day, how long would 6 men need to build a wall 10 m long if they worked 5 hours a day?

Solution (the required dimension is at the end of the statement):

$$5 \text{ M} \rightarrow 8 \text{ m} \rightarrow 7 \text{ h} \rightarrow 12 \text{ d}$$
$$6 \text{ M} \rightarrow 10 \text{ m} \rightarrow 5 \text{ h} \rightarrow x \text{ d}$$

1. Reduction to unit 1 man

The following text is not usually written down, but should always be spoken when formulating the equation, in roughly these words:

If 5 men take 12 days at 7 hours a day for 8 metres, does 1 man need more days or less for 8 metres at 7 hours a day?

It is important always to ask the question whether the unknown quantity will be more or less. Here, 1 man takes 5 times as many days as 5 men.

$$1 \text{ M} \rightarrow 8 \text{ m} \rightarrow 7 \text{ h/d} \rightarrow 12 \times 5 \text{ days}$$

2. Reduction to the unit 1 metre

The statement is the conclusion reached above.

If 1 man takes 12 x 5 days for 8 metres at 7 hours per day, does he need more days or less than 12 x 5 for 1 metre at 7 hours per day?

Answer: he needs one-eighth of the time.

Conclusion: 1 M → 1 m → 7 h/day → $\frac{12 \times 5}{8}$ days

3. Reduction to the unit 1 hour per day.

The statement is again the preceeding conclusion.

If 1 M takes $\frac{12 \times 5}{8}$ days for 1 m at 7 h/days,

1 man will take 7 times as long to build 1 m at only 1 hour per day.

$$1\ M \rightarrow 1\ m \rightarrow 1\ h/day \rightarrow \frac{12 \times 5 \times 7}{8}\ days$$

The unit has now been calculated for each dimension; from these the multiples required by the problem must be calculated. It is advisable to do this for each dimension separately.

$$6\ M \rightarrow 10\ m \rightarrow 5\ h/day = x\ days$$

If 1 man takes $\frac{12 \times 5 \times 7}{8}$ days at 1 h/d,

do 6 men need more or less days for the same work?

Answer: They need one-sixth of the time.

$$6\ M \rightarrow 1\ m \rightarrow 1\ h/day \rightarrow \frac{12 \times 5 \times 7}{8 \times 6}$$

This method is used to calculate whatever multiple is required by the problem.

The example may be written in abbreviated form as follows:

$$5\ M \rightarrow 8\ m \rightarrow 7\ h/day \rightarrow 12\ days$$
$$\underline{6\ M \rightarrow 10\ m \rightarrow 5\ h/day \rightarrow x\ days}$$

Units:

$$1\ M \rightarrow 8\ m \rightarrow 7\ h/day \rightarrow 12 \times 5$$
$$1\ M \rightarrow 1\ m \rightarrow 7\ h/day \rightarrow \frac{12 \times 5}{8}$$
$$1\ M \rightarrow 1\ m \rightarrow 1\ h/day \rightarrow \frac{12 \times 5 \times 7}{8}$$

Multiples:

$$6\ M \rightarrow 1\ m \rightarrow 1\ h/day \rightarrow \frac{12 \times 5 \times 7}{8 \times 6}$$
$$6\ M \rightarrow 10\ m \rightarrow 1\ h/day \rightarrow \frac{12 \times 5 \times 7 \times 10}{8 \times 6}$$

$$6\ M \rightarrow 10\ \ m \rightarrow 5\ \ h/day \rightarrow \frac{12 \times 5 \times 7 \times 10}{8 \times 6 \times 5}$$

It is advisable not to simplify the fraction until the end. The complete fraction is:

$$x = \frac{12 \times 5 \times 7 \times 10}{8 \times 6 \times 5} = \frac{70}{4} = \frac{35}{2} = 17.5 \text{ days}$$

Thus to build a wall 10 m long, working at a rate of 5 hours per day, 6 men need a total of 17.5 days.
The number of men, the length of the wall and the number of hours worked per day are altered in sucession - whereby the order in which this is done is unimportant. It is advisable, however, to keep to the order chosen for the conditional statement in the first operation.

Simple proportion
Sometimes problems on proportion are so simple that they can be solved without first determining the unit, as is normally necessary in applications of the rule of three.
Note:
The rule-of-three method may only be used if twice, 3 times, 4 times ect. one quantity is equal to twice, 3 times, 4 times etc. or else half, one-third, one-fourth, etc. of the other.

3.2 Percentages

A percentage is basically a ratio in which the second number is arranged to be 100. Thus

$$1\% = \frac{1}{100} \text{ and } = 0.01$$

1% of a quantity is therefore $\frac{\text{original amount}}{100}$

If a particular percentage of the whole is to be determined, the multiple, i.e. the percentage value, must be calculated from the unit, 1%.

Example: $1\% = \frac{\text{original amount}}{100}$

$$20\% = \frac{\text{original amount}}{100} \times 20$$

The rule is thus

$$\text{Percentage value} = \frac{\text{original amount} \times \text{percentage rate}}{100}$$

$$P = \frac{G \times p}{100}$$

200%, 300% etc. indicate double, three times etc. the original amount.

Examples:

Of a total of 400 m^3 of water, 68 m^3 are lost due to a burst pipe. What percentage does this represent?

400 m^3 100%

68 m^3 x

1 m^3 $\frac{100\%}{400}$

68 m^3 $\frac{100 \times 68}{400} = 17\%$

Rule:

$$p\% = \frac{100 \times P}{G}$$

A special type of problem is one in which the percentage rate and the increased or reduced percentage value are given and the original amount sought.

Example:

Due to a 15% increase in the delivery rate, 460 m^3 of water are now discharged. What was the original amount?

Solution: 100% + 15% = 460 m^3

100% = x m^3

115% - 460 m^3

1% - $\frac{460 \text{ m}^3}{115}$

$$\text{Original amount } 100\% = \frac{460 \times 100}{115} = 400 \text{ m}^3$$

General rule:

With increased amount, $G = \frac{(G + P) \times 100}{100 + p}$

With reduced amount, $G = \frac{(G - P) \times 100}{100 - p}$

4 Introduction to elementary algebra

4.1 Working with algebraic symbols

Draw a rectangle. Suppose its length to be 4 m and its width 3 m. The area is then 4 m x 3 m = 12 m^2 . The numbers "four" and "three" are specific numbers, the figures "4" and "3" are specific numerals. The general formula for finding the area of a rectangle can be expressed as A = l x w, whereby the letters may represent any quantities. The letters are known as algebraic symbols and with their aid mathematical formulae and rules of arithmetic can be written down in a universally applicable way. The same basic rules apply in algebra as in arithmetic.

Addition and subtraction

Example: $a + c + a - b = a + a - b + c = 2a - b + c$

Only like terms may be added or subtracted. The multiplication sign between the number and the letter may be omitted, i.e. 2a instead of 2 x a (the multiplication sign is avoided as far as possible in algebra since it may cause confusion).

Example: $18a + a - 5a = 14a$

$8a + 2a + 3b = 10a + 3b$

Multiplication and division

When multiplying expressions with letters and soefficients, e.g. 5a x 2b, the coefficients are multiplied together (5 x 2 = 10) and the letters written after the resulting figure in alphabetical order. The multiplication sign may be omitted between the letters and between coefficient and letter, but must be written between the numerical factors, e.g. 12 x 3b = 36b.

In division of expressions, coefficients and letters are simplified as far as possible.

Example:

$$\frac{8c}{4a} = \frac{2c}{a}; \quad \frac{6a \times 3c}{2c} = \frac{\not{2} \times 3a \times 3\not{c}}{\not{2}\not{c}} = 9a$$

If a sum or a difference is to be divided by a certain number, every term in the sum or difference must be divided by this number.

$$\frac{a+b}{b} = \frac{a}{b} + \frac{b}{b} = \frac{a}{b} + 1$$

$$\frac{c-d}{c} = \frac{c}{c} - \frac{d}{c} = 1 - \frac{d}{c}$$

In algebraic calculations, the same rule applies as in simple arithmetic: multiplication or division come before addition or subtraction.

4.2 Use of brackets

In paragraph 1.1.6 we learnt the rule that multiplication or division must be carried out before addition or subtraction. If, however, in a specific case, an addition or subtraction operation must be performed before multiplication or division, this is indicated by placing the relevant terms inside brackets, e.g.:

$$(2 + 4) \times 5 = 6 \times 5 = 30$$

$$\frac{24}{(10 - 8)} = \frac{24}{2} = 12$$

The figures in brackets are worked out first.

Factorisation

Example: What overall length results from addition of the partial lengths?

$$\overset{a}{\boxed{85}} + \overset{b}{\boxed{150}} + \overset{c}{\boxed{60}} + \overset{b}{\boxed{150}} + \overset{a}{\boxed{85}} + \overset{c}{\boxed{60}}$$

Solution: 2 x a + 2 x b + 2 x c

2 x 85 + 2 x 150 + 2 x 60

170 + 300 + 120

The individual products have a common factor which can be put in front of the brackets:

2 x (a + b + c) or 2 x (85 + 150 + 60)

Expansion of brackets - multiplying out

(a + b) x c = ac + bc

Each term inside the brackets is multiplied by the quantity outside the brackets.

\+ in front of the brackets

a + (b - c) = a + b - c

If a + sign comes before the brackets, the sign of all terms is unaltered after removal of the brackets.

7 + (8 - 4) = 7 + 4 = 11

\- in front of the brackets

a - (b - c) = a - b + c

On removal of the brackets, the signs of all terms inside the brackets are changed.

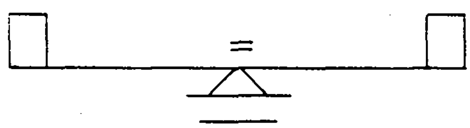
15 - (6 + 8) = 1 or 15 - 6 - 8 = 1

14 14

4.3 Equations: principles and basic rules

An equation can be compared with a pair of scales which are in a permanent position of balance. It consists of three parts:

1. Left-hand side of the equation
2. Right-hand side of the equation
3. Equals sign



If any alterations are carried out, the same operation must always be performed on both sides of the equation so that the balance is not upset.

Rule: If a term is transferred from one side of an equation to the other side, the sign changes.

$y + 5 = 9$

$y + 5 - 5 = 9 - 5$

$y = 9 - 5$

The unknown quantity is y. If y is to stand alone, 5 must be subtracted from the left-hand side of the equation. This operation must be repeated on the right-hand side.

+ becomes -

$y - 6 = 9$

$y - 6 + 6 = 9 + 6$

$y = 15$

- becomes +

$y \times 5 = 15$

$y \times \frac{5}{5} = \frac{15}{5}$

$y = 3$

x becomes ÷

$\frac{y}{3} = 4$

$\frac{y \times 3}{3} = 4 \times 3$

$y = 12$

: besomes x

Note also that the two sides of the equation are interchangeable.

Example: $3 + 2 = 5$

$5 = 3 + 2$

4.4. Isolation of the unknown quantity

Every formula is basically an equation, e.g. $A = l \times b$

This expresses two things:

1. What is to be calculated (A)
2. How it is to be calculated ($l \times b$)

A formula always consists of two or more known quantities and one unknown quantity.
The formula (equation) is often arranged in such a way that the unknown quantity (A) stands alone on the left of the equals sign. Its value can only be found if it stands alone. If this is not the case, the formula must be re-arranged so that the unknown quantity is isolated.

Application of this rule to the solution of a problem.
Method of solution:

1st step: Finding the basic formula for the problem.

Example: $V = \frac{d^2 \times \pi \times h}{4}$ (cm^3)

2nd step: Determination of the unknown quantity.

$V = \frac{d^2 \times \pi \times h}{4}$ (cm^3)

3rd step: If applicable, removal of the fraction b² multiplying both sides by the denominator of the fraction.

$4 \times V = \frac{d^2 \times \pi \; h \times 4}{4}$

4th step: Transposition of the unknown quantity to the left-hand side of the equation.

$d^2 \times \pi \times h = 4V$

5th step: Isolation of the unknown quantity.

$$\frac{d^2 \times \pi \times h}{d^2 \times \pi} = \frac{4\ V}{d^2 \times \pi}$$

$$h = \frac{4\ V}{d^2 \times \pi}$$

6th step: Substitution of numbers

5 Units of measurement

5.1. Time and angle measurement

The methods used to measure time and angles are comparable because both are based on the number 60. Time es expressed in terms of seconds (s), minutes (min) and hours (h).

1 h = 60 min = 60 x 60 s = 3600 s

Angles are given in degrees (^{o}), minutes (') and seconds (").

1^{o} = 60' = 60 x 60" = 3600"

Angles can be added together, subtracted, multiplied and divided in the same way as time.

Example 1: Addition

```
    56 h 32 min 45 s
+   63 h 24 min 17 s
   119 h 56 min 62 s
   119 h 57 min  2 s
```

Example 2: Subtraction

```
               61 h 34 min 42 s
             - 38 h 36 min 27 s
Convert        61 h 34 min 42 s
             = 60 h 94 min 42 s
Subtract     - 38 h 36 min 27 s
             = 22 h 58 min 15 s
```

Example 3: Multiplication

```
62 h 34 min 56 s x 5
62 h      x 5  = 310 h
34 min    x 5  =          170 min
56 s      x 5  =                  280 s
               = 310 h    170 min 280 s
               = 312 h     54 min  40 s
```

Example 4: Division

```
33 h 17 min 28 s ÷ 4 = 8 h 19 min 22 s
32 h
 1 h = 60 min
     + 17 min
       77 min ÷ 4
       76 min
        1 min = 60 s
              + 28 s
                88 s ÷ 4
```

Example 5: Conversion

How many minutes are there in
2 h 24 min 36 s?

2 h = 2 x 60 = 120 min
24 min = 24 min
36 s = 36 : 60 = 0.6 min
= 144.6 min

How many hours, minutes and seconds are there in 33.28 h?

33 h = 33 h
0.28 h = 0.28 x 60 = 16.8 min
0.8 min = 0.8 x 60 = 48 s
33.28 h = 33 h 16 min 48 s

5.2. Linear, square and cubic measure

There are two basic systems of measurement:

a) The metric system
The unit of length, one metre (1 m), is based on the light wavelength. 1 m is defined as 1,650,763.73 times the wave-lenght of krypton gas in the spectrum.

b) The Britisch Imperial system, with the inch as the smallest unit. One inch is equivalent to 25.4 mm.

Metric measure	Symbol	Imperial measure	Symbol
Metre	m	Yard	yd
Decimetre	dm	Foot	ft (')
Centimetre	cm	Inch	in (")
Millimetre	mm		
Micrometre	µm		
Kilometre	km		

Square measure	Symbol	Cubic measure	Symbol
Square metre	m^2	Cubic metre	m^3
Square decimetre	dm^2	Cubic decimetre	dm^3
Square centimetre	cm^2	Cubic centimetre	cm^3
Square millimetre	mm^2	Cubic millimetre	mm^3
Square kilometre	km^2	Litre	l
Hectare	ha	Hectolitre	hl

Conversion

Lager unit ← Conversion to → Smaller unit

135.4 mm = ? cm	1 power	23.5 m = ? dm
135.4 mm = 13.54 cm	linear	23.5 m = 235 dm
135.4 mm^2 = ? cm^2	2 powers	23.5 m^2 = ? dm^2
135.4 mm^2 = 1.354 m^2	square	23.5 m^2 = 2350 dm^2
	quantity ^{2}	
135.4 mm^3 = ? cm^3	3 powers	23.5 m^3 = ? dm^3
135.4 mm^3 = 0.1354 cm^3	cubic	23.5 m^3 = 23500 dm^3
	quantity^3	

5.3 Calculation of mass

The unit of mass is the kilogram (kg). Derived units are the tonne (t), gram (g) and milligram (mg). The conversion factor from one unit to the other is 1000.

1 t = 1000 kg = 1,000,000g = 1,000,000,000 mg

The mass per unit volume (of 1 cm^3, 1 dm^3 or 1 m^3) is calles density.

$$\frac{g}{cm^3} \text{ or } \frac{kg}{dm^3} \text{ or } \frac{t}{m^3} = \frac{\text{mass}}{\text{volume}} = \text{density}$$

Mass = volume x density
m = V x d

5.4 Calculation of weight

The weight of a body is the force pulling it towards the earth and with which it presses on the surface on which it rests.

Symbols:

F = force (n)

m = mass (kg)

g = acceleration of the earth $\frac{m}{s^2}$)

$$F = m \times g$$

A mass of 1 kg exerts a force of

$1 \text{ kg} \times 9.81 \frac{m}{s^2} = 9.81 \frac{kg\ m}{s^2} = 9.81 \text{ N} \quad 10\text{N}$

6 Speed

6.1 Uniform velocity

Velocity is the rate of change of distance moved with time.

Notations:

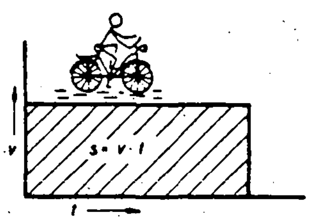
v = velocity (km/h, m/min, m/s)

s = distance (km, m, cm, mm)

t = time (h, min, s)

d = diameter (mm)

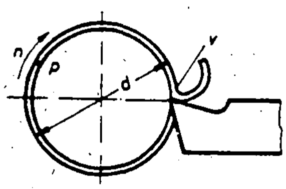
n = rotational speed (1/min)



Velocity in a straight line

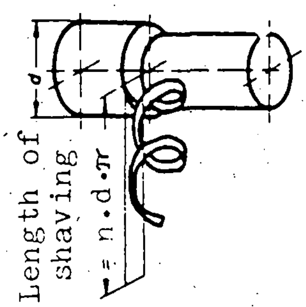
$$v = \frac{s}{t} \qquad \frac{km}{h}, \quad \frac{m}{min}, \quad \frac{m}{s}$$

In the velocity/time graph, the distance appears as an area.



Angular velocity

The path covered in one revolution, $s = d \times \pi$, is equivalent to the circumference of the circle. The path covered in n revolutions is equivalent to $d \times \pi \times n$.



If the "n" revolutions are completed in a specific time, e.g. per minute, there results a rotational speed.

$$v = d \times \pi \times n \left[\frac{m}{min}\right]$$

If the diameter is given in mm, the result must be divided, by 1000.

6.2 Average speed

In machine tools, e.g. scroll saws, a rotary motion is converted into a longitudinal lifting motion by means of a crank mechanism. In contrast, in engines a rotary motion is produced from the longitudinal motion - as in the engine of a motor-car. In all cases, the rotary motion is uniform and the longitudinal motion non-uniform.

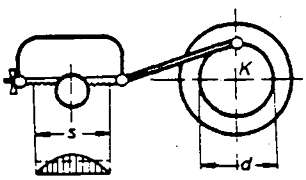
Notations:

v_m = average speed (m/min, m/s)

n = rotational or stroke speed (1/min)

s = distance, lenght of stroke (m)

Lifting speed



At 1 revolution from point K, the distance is 2 x s, at "n" revolutions from point K, the distance is 2 x s x n. At n revolutions per min,

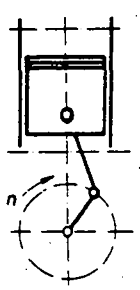
$$v_m = 2 \cdot x\ s\ x\ n \left[\frac{m}{min}\right]$$

If the length of the stroke is given in mm,

$$v_m = \frac{2 \times s \times n}{1000} \left[\frac{m}{min}\right]$$

Piston speed

The piston moving backwards and forwards changes its position constantly between top dead centre and bottom dead centre. The same average speed v_m therefore also applies here.



$$v_m = 2 \times s \times n \left[\frac{m}{min}\right]$$

If the piston speed is to be determined in m/s,

$$v_m = \frac{2 \times s \times n}{60} = \frac{s \times n}{30} \left[\frac{m}{s}\right]$$

7 Belt drives, gear units, worm drives

7.1 Simple belt drive

d_1 = diameter of driving wheel

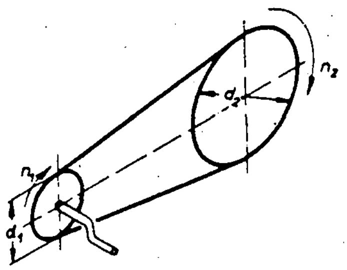
n_1 = rotational speed of driving wheel

d_2 = diameter of driven wheel

n_2 = rotational speed of driven wheel

i = transmission

Cirumferential velocity



The circumferential velocity of the driving wheel is

$$v_1 = d_1 \times \pi \times n_1$$

The circumferential velocity of the driven wheel is

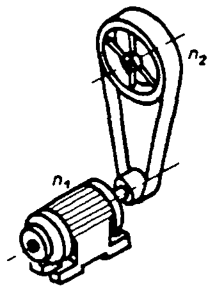
$$v_2 = d_2 \times \pi \times n_2$$

Since the rotating belt has only one circumferential velocity, then without slip

$$v_1 = v_2$$

$$d_1 \times \not{\pi} \times n_1 = d_2 \times \not{\pi} \times n_2$$

Transmission ratio



This is the ratio of the speed of the driving wheel to that of the driven wheel.

$$i = \frac{\text{driving speed}}{\text{driven speed}} = \frac{\text{driven diameter}}{\text{driving diameter}}$$

$$i = \frac{n_1}{n_2} = \frac{d_2}{d_1}$$

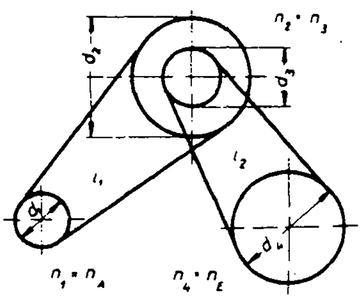
The ratio of speeds is the reciprocal of the ratio of diameters.

7.2. Multiple belt drive

n_A = starting speed of driving wheel
n_E = end speed
i_1 = first partial transmission
i_2 = second partial transmission
i = total transmission

The odd index numbers 1, 3 etc. always stand for driving elements.

Partial transmissions



The complete drive is divided into separate components and each partial transmission determined.

Paterial transmission 1

driving d x n = driven d x n

$d_1 \times n_1 = d_2 \times n_2$

$$i_1 = \frac{n_1}{n_2} = \frac{d_2}{d_1}$$

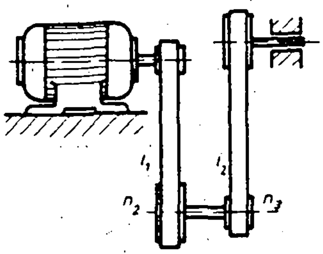
Partial transmission 2

driving d x n = driven d x n

$d_3 \times n_3 = d_4 \times n_4$

$$i_2 = \frac{n_3}{n_4} = \frac{d_4}{d_3}$$

Total transmission



Every total transmission is the product of the partial transmissions.

$i = i_1 \times i_2$

$$i = \frac{n_1 \times n_3}{n_2 \times n_4} \text{ or } \frac{d_2 \times d_4}{d_1 \times d_3}$$

$$\frac{n_1}{n_2} = \frac{\text{starting speed}}{\text{end speed}} = \frac{\text{driven dia.}}{\text{driving dia.}}$$

Wheels with a common shaft run at the same speed, therefore $n_2 = n_3$.

7.3 Gear wheel measurement

d = diameter of pitch circle

d_k = diameter of addendum circle

d_f = diameter of denendum circle

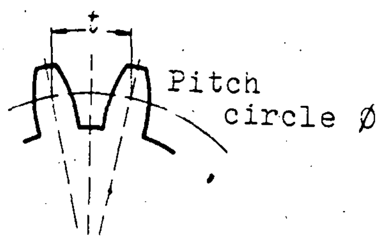
t = pitch

m = module

h = tooth depth = $h_f + h_k$

h_f = dedendum = $7/6 \times m$

h_k = addendum $\hat{=}$ module



Pitch

The pitch is the centre-to-centre distance between successive teeth, measured along the pitch circle. The circumference of the pitsch circle can be used to calculate t:

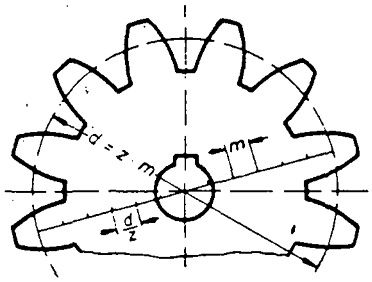
$$0 = t \times z \qquad 0 = d \times$$

$$t \times z = d \times \pi \longrightarrow \boxed{t = \frac{d \times \pi}{2}}$$

Module

From the relationship $d \times \pi = t \times z$, the ratio d/z can also be expressed by t/π :

$$\boxed{\frac{d}{z} = \frac{t}{\pi} = m}$$



The module was introduced as a dimension to represent these equivalent ratios. The module is given in mm. Module 1 is equivalent to a pitch of 3.14159.... mm (π), measured along the pitch circle. For measurements along the radius, however, module 1 = 1 mm.

The module is a standardized quantity which is used to enable calculations to be carried out with whole numbers.

7.4. Simple gear mechanism

z = number of teeth

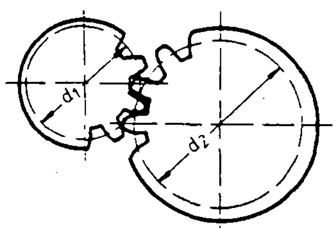
n = speed

i = transmission

a = centre distance

d = diameter of pitch circle

Relationship between d and n



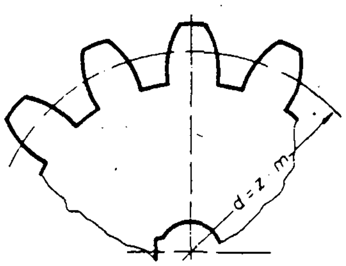
Since the gear wheels intermesh, they must rotate with the same circumferential velocity.

$$v_1 = v_2$$

$$d_1 \times \pi \times n_2 = d_2 \times \pi \times n_2$$

$$d_1 \times n_1 = d_2 \times n_2$$

Relationship between z and n



In the equation $d_1 \times n_1 = d_2 \times n_2$,

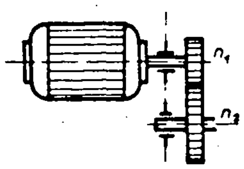
d is replaced by z x m:

$$z_1 \times m \times n_1 = z_2 \times m \times n2$$

$$\boxed{z_1 \times n_1 = z_2 \times n_2}$$

Driving z x n = driven z x n

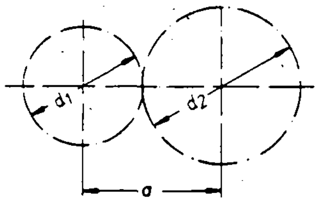
Transmission ratio



If the speed ratio is determined by the driving wheel, then

$$i = \frac{\text{driving speed}}{\text{driven speed}} = \frac{n_1}{n_2} \quad \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

Centre distance



The centre distance is given by the diametres d_1 and d_2.

$$a = \frac{d_1 + d_2}{2} = \frac{z_1 \times m + z_2 \times m}{2}$$

$$= \frac{m}{2} \times (z_1 + z_2)$$

7.5 Multiple gear mechanism

n_A = starting speed

n_E = end speed

i_1 = first partial transmission

i_2 = second partial transmission

i = tataltransmission

The odd index numbers 1, 3 etc. stand for driving wheels.

Partial transmissions

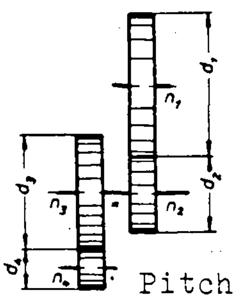
We divide each drive up according to the basic rule

driving d x n = driven d x n

driving z x n = driven z x n

Partial transmission 1

$$d_1 \times n_1 = d_2 \times n_2, \; z_1 \times n_1 = z_2 \times n_2$$



$$i_1 = \frac{n_1}{n_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

Pitch circle Ø

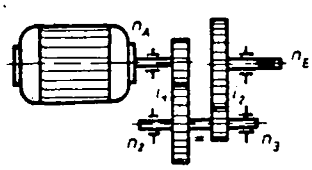
Partial transmission 2

$d_3 \times n_3 = d_4 \times n_4$, $z_3 \times n_3 = z_4 \times n_4$

$$i_2 = \frac{n_3}{n_4} = \frac{d_4}{d_3} = \frac{z_4}{z_3}$$

Total transmission

The total transmission is the product of the partial transmissions.



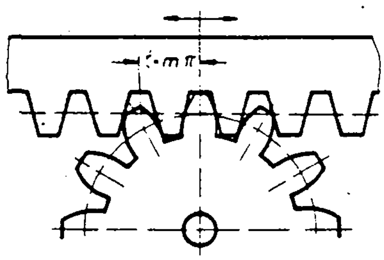
$i \quad i_1 + i_2$

$$i = \frac{n_1 \times n_3}{n_2 \quad n_4} = \frac{d_2 \times d_4}{d_1 \quad d_3} = \frac{z_2 \times z_4}{z_1 \quad z_3} = \frac{n_A}{n_E}$$

7.6. Rack-and pinion gearing

s = rack travel = angle of deflection

Rack travel



The distance travelled by the rack is determinded by the circumference of the pitch circle.

Rack travel = pitch-circle circ.

s d x 3.14 } for 1 revolution
s = z x t

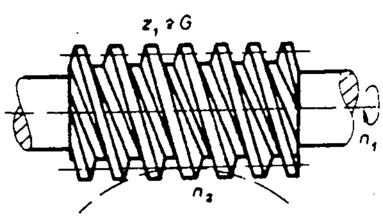
$$s = z \times t \times \frac{\alpha}{360°}$$ → 1 partial rev.

7.7. Worm drive

G = number of worm threads (z_1)

z_2 = number of teeth of the worm wheel

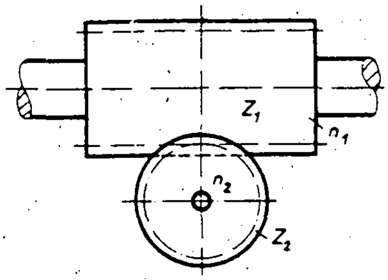
Threading



If a single-threaded worm is turned once, the worm wheel moves forward by one tooth.

Number of threads = number of teeth

Relationships



The relationships are the same as in the case of simple gears:

Driving z x n = driven z x n

$$z_1 \times n_1 = z2 \times n_2$$

z_1 is replaced by G = number of threads

$$i = \frac{n_1}{n_2} = \frac{z_2}{G}$$

In the case of multi-threaded worms, pitch is equal to spacing times the number of threads.

8 Work, power, efficiency

F = force in N

s = distance in m

t = time in s

v = speed in m/s

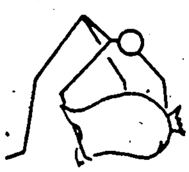
W = mechanical work in J

P = power in W

P_{ef} = work output

P_{in} = work input

Work



Work = force x distance

W = F (in N) x s (in m)

The unit of work is the joule (J).

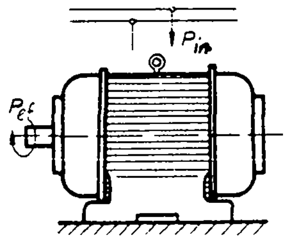
1 joule is the work done when the point of application of a force of 1 N moves trought 1 m in the direction of the force.

1 Nm ≘ 1 J ≘ 1 Ws

Power


Power = work per unit of time
$P = \frac{W}{t}$ (in $\frac{Nm}{s}$) $= \frac{J}{s} = \frac{Ws}{s}$
Power = force x speed
$P = \frac{W}{t} = \frac{F \times s}{t} = F \times v$

Efficiency (Etá)



$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}} = \frac{\text{useful work}}{\text{effort}}$$

$$\eta = \frac{P_{ef}}{P_{in}}$$

9 Basic electrical quantities

V = potential difference (voltage), measured in volts (V)

I = current, measured in amperes (A)

R = resistance, measured in ohms (Ω)

For a better understanding of these basic quantities, imagine the discharge valve of a water pipe which is under pressure.

Voltage	≙	electron pressure	≙	water pressure
Current	≙	electron flow	≙	water volume
Resistance	≙	electron obstruction	≙	throttling

An increase of the potential difference at constant resistance simultaneously increases the current.

An increase of resistance at a constant potential difference simultaneously reduces the current.

9.1 Ohm's law

Ohm's law is derived from the two relationships I V and $I \sim 1/R$.

$$V \sim I \quad \frac{1}{R} \sim I \quad = \frac{V}{R} \qquad \boxed{V = R \times I}$$

Ohm's law is valid for direct current and for alternating current only under ohmic loading.

9.2. Resistance

R = resistance (Ω)

A = area of cross-section (mm^2)

L = sum of the lengths of bridge wire (m)

ρ = resistivity (Ω x mm^2/m)

κ = electrical conductivity (m/Ω x mm^2

Resistance is dependent on material, length, area of cross-section and temperature.

The resistivity of a material is numerically equal to the resistance of a conductor made of the material, of length 1 metre and area of cross-section 1 mm^2 at 20°C.

For Cu, ρ = 0.0178 ohms.

The greater the resistivity of a material, the higher the resistance to conduction.

Hence: $R \sim \rho$

The longer the wire, the higher the resistance.

Hence: $R \sim L$

The greater the area of cross-section, the smaller the resistance.

Hence: $R \sim \frac{1}{A}$

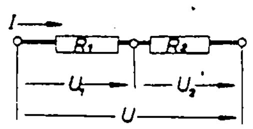
From these three relationships there results the equation

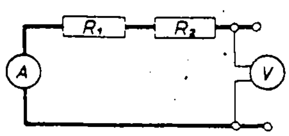
$$R = \frac{\rho \times L}{A} \text{ or } \frac{L}{\kappa \times A}$$

ρ = ro
κ = kappa

9.3 Connection of resistors

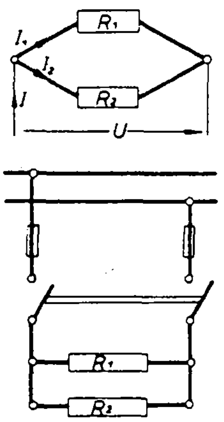
Resistors in series





The same current I flows through each connector consecutively. Each resistor requires an individual voltage. The total resistance of resistors connected in series is the sum of the individual resistances.

Resistors in parallel



The same voltage V is applied to all resistors place side by side with their corresponding ends joined together.

$V = V_1 = V_2 = \text{constant}$

Each resistor requires an individual current.

$I = I_1 + I_2 + \ldots.$

The combined resistance of resistors in parallel is always less than the lowest individual resistance.

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \qquad \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

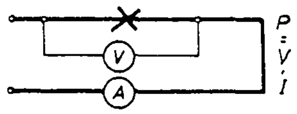
9.4. Electric power and energy

P = power (W, kW)

W = work (energy) (Wh, kWh)

t = time (h)

Electric power



If we increase the current at constant voltage, the brightness, i.e. power of a light-bulb increases.

Power = voltage x current

The unit of electric power is the watt (W). 1 watt is a rate of transfer of energy of 1 joule per second. Thus

1 Nm/s = 1 J/s = 1 W

Electric energy

Just as in mechanics, the rule is:

Power = work/time, thus:

Work (energy) = power x time

W = P x t

The unit of electric energy is the watt second (Ws) Ohne watt second is the derived unit of mechanical work.

1 Nm = 1 J = Ws

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TRAINING MODULES FOR WATERWORKS PERSONNEL

List of training modules:

Basic Knowledge

- **0.1** Basic and applied arithmetic
- **0.2** Basic concepts of physics
- **0.3** Basic concepts of water chemistry
- **0.4** Basic principles of water transport
- **1.1** The function and technical composition of a watersupply system
- **1.2** Organisation and administration of waterworks

Special Knowledge

- **2.1** Engineering, building and auxiliary materials
- **2.2** Hygienic standards of drinking water
- **2.3a** Maintenance and repair of diesel engines and petrol engines
- **2.3b** Maintenance and repair of electric motors
- **2.3c** Maintenance and repair of simple driven systems
- **2.3d** Design, functioning, operation, maintenance and repair of power transmission mechanisms
- **2.3e** Maintenance and repair of pumps
- **2.3f** Maintenance and repair of blowers and compressors
- **2.3g** Design, functioning, operation, maintenance and repair of pipe fittings
- **2.3h** Design, functioning, operation, maintenance and repair of hoisting gear
- **2.3i** Maintenance and repair of electrical motor controls and protective equipment
- **2.4** Process control and instrumentation
- **2.5** Principal components of water-treatment systems (definition and description)
- **2.6** Pipe laying procedures and testing of water mains
- **2.7** General operation of water main systems
- **2.8** Construction of water supply units
- **2.9** Maintenance of water supply units Principles and general procedures
- **2.10** Industrial safety and accident prevention
- **2.11** Simple surveying and technical drawing

Special Skills

- **3.1** Basic skills in workshop technology
- **3.2** Performance of simple water analysis
- **3.3a** Design and working principles of diesel engines and petrol engines
- **3.3b** Design and working principles of electric motors
- **3.3c** –
- **3.3d** Design and working principle of power transmission mechanisms
- **3.3e** Installation, operation, maintenance and repair of pumps
- **3.3f** Handling, maintenance and repair of blowers and compressors
- **3.3g** Handling, maintenance and repair of pipe fittings
- **3.3h** Handling, maintenance and repair of hoisting gear
- **3.3i** Servicing and maintaining electrical equipment
- **3.4** Servicing and maintaining process controls and instrumentation
- **3.5** Water-treatment systems: construction and operation of principal components: Part I - Part II
- **3.6** Pipe-laying procedures and testing of water mains
- **3.7** Inspection, maintenance and repair of water mains
- **3.8a** Construction in concrete and masonry
- **3.8b** Installation of appurtenances
- **3.9** Maintenance of water supply units Inspection and action guide
- **3.10** –
- **3.11** Simple surveying and drawing work

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